Structures- Unit 3 Physics

Main Concepts to cover

3-OPTION	**Study Design 2009 – 2012** **Unit 3 Detailed Study: Materials and their use in structures**
	identify different types of external forces (compression, tension and shear) which can act on a body, including gravitational forces
	evaluate the suitability of different materials for use in structures (including beams, columns and arches) by comparing tensile and compressive strength and stiffness **or flexibility** under load
	analyse the behaviour of materials under load in terms of extension and compression (including Young's modulus; $Y = \frac{\sigma}{\varepsilon}$)
	calculate the stress and strain resulting from the application of forces and loads to materials in structures; ($\sigma = \frac{F}{A}$, $\varepsilon = \frac{\Delta l}{l}$)
	describe brittle and ductile failure and **apply** data to predict brittle or ductile failure under load
	calculate the potential energy stored in a material under load (strain energy) using area under stress versus strain graphs
	evaluate the toughness of a material tested **to the point of failure**,
	describe elastic or plastic behaviour of materials under load and the resulting energy **transformed to** heat
	evaluate the suitability of a composite material **for its use** in a structure by considering its properties and the properties of the component materials (maximum of three components);
	calculate torque, $\tau = r \perp F$
	analyse translational and rotational forces (**torques)** in **simple** structures (**including uniform** columns**, struts, ties**, beams, cables; **not including trusses**) modelled as two dimensional structures in **static** equilibrium
	Identify and apply safe and responsible practices when working with structures, materials and associated measuring equipment **in investigations of materials.**

This unit we will be examining how materials behave when under force and we will examine the stability and twisting forces involved in various structures. Also we will examine the loads that bridges experience and tie it back to the physics of structures.

Let us make a few definitions to get us started:

$$mm \xrightarrow{\div 10} cm \xrightarrow{\div 100} m \xrightarrow{\div 1000} km$$

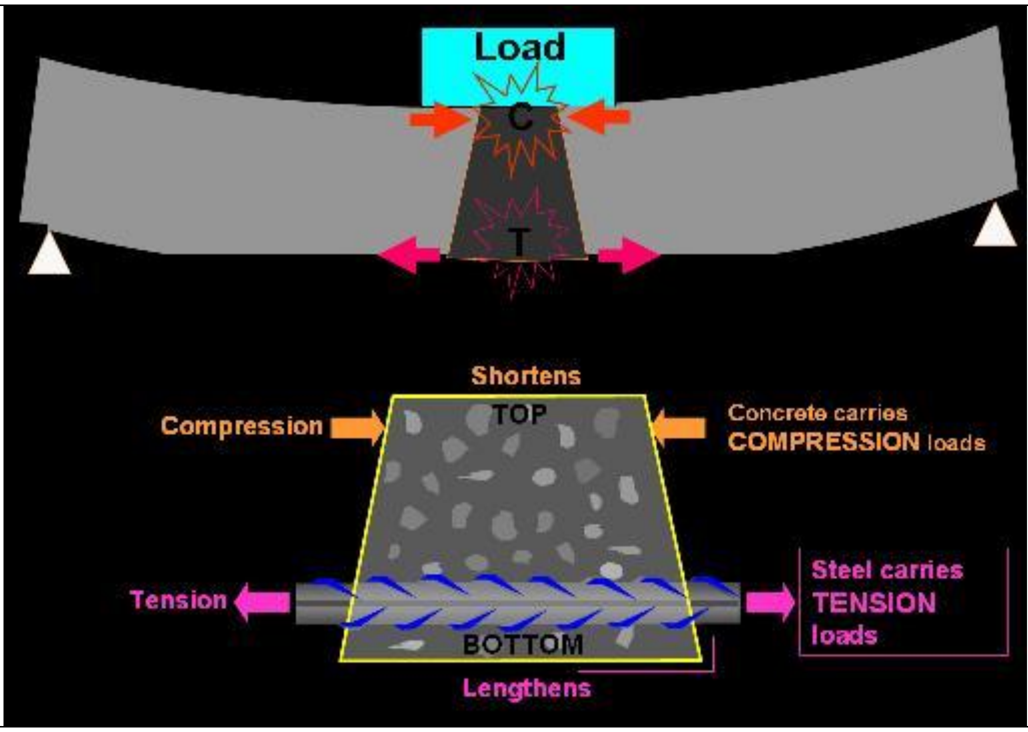
$$km \xrightarrow{\times 1000} m \xrightarrow{\times 100} cm \xrightarrow{\times 10} mm$$

Converting area

$$mm^2 \xrightarrow{\div 100} cm^2 \xrightarrow{\div 10,000} m^2 \xrightarrow{\div 10,000} ha \xrightarrow{\div 100} km^2$$

http://www.pbs.org/wgbh/buildingbig/lab/materials.html

Definitions	What is it	How is it used
Load	Forces a object experiences	It is basically the weight of the object – remember this is a force
Metric definitions	n = nano = 10^{-9} µ = micro = 10^{-6} m = milli = 10^{-3} k = kilo = 10^{3} M = mega = 10^{6} G = giga = 10^{9} 1 tonne = 10^{3} kg	

<table>
<tr><td>Compression
Tension</td><td colspan="2">
</td></tr>
<tr><td>Compression</td><td colspan="2">Atoms are pushed together</td></tr>
<tr><td>Tension</td><td>Atoms are pulled apart</td><td></td></tr>
<tr><td>Loads</td><td>Forces that act on sturctures are called loads</td><td></td></tr>
<tr><td>Stress-Defined as the force per unit area of cross sectional supporting the load</td><td>$\sigma = \frac{F}{A}$

F= force in Newtons
A= cross sectional area , must be in metres squared</td><td>We use stress to compare the strength of materials- a strong material will able to experience a greater stress than a weaker materials.
Stress measure in Pascals ($1Pa = 1Nm^{-2}$)
Materials experience huge stress values such as megapascalls , MPa where 1 $MPa = 1\times10^{6} Pa$

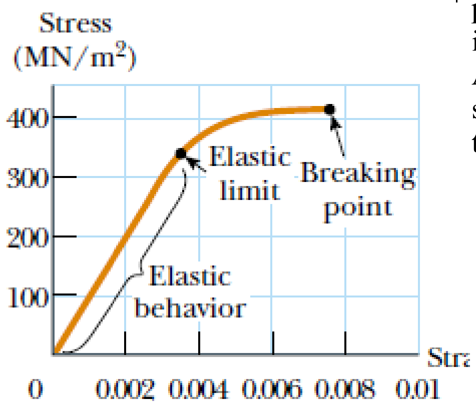
All materials can only handle up to certain value, after this value the materials deforms</td></tr>
<tr><td>Problem</td><td>A 10 mm wire can support a stress of 250 MPa safely. If a person uses this wire to support a mass of 1200 kg, will the wire be able strong enough?</td><td>Lets us work out the stress the wire will experience

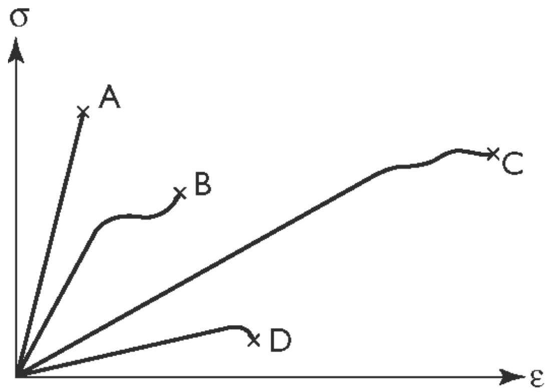
Warning : 10mm diameter is not in metres, so it is best to convert
radius = 5 mm = 0.005m
Area of crossectional =
$\pi r^2 = 3.14\times(.005)^2 = 7.85\times10^{-5}$

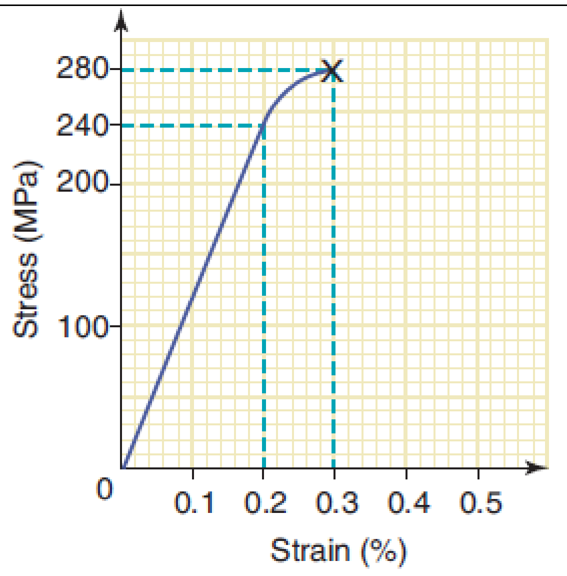
Force it will experience is
$W = mg = 1200\times10 = 12000N$

So stress the wire will experience is then
$\sigma = \frac{F}{A} = \frac{12000}{7.85\times10^{-5}} = 152866242 = 152.866MI$

Since the wire can support a stress of 250 MPa ,</td></tr>
</table>

		the wire should be able to support a mass of 1200 kg
The result of stress is the strain a material experiences Strain , symbol ε	$\varepsilon = \frac{\Delta l}{l}$ $\Delta l =$ change of length $l =$ original length	Strain is a measure of the magnitude of this deformation. The amount a material deforms depends on its initials length in the direction of the load. No units (but remember to make sure the change of length and the original length are in the same units.) The strain of a material when it breaks is expressed as a percentages and called percentage elongation.
Problem	A 3m long cable is stretched to 3.2 m when it is used to lift a heavy object. a)What is the strain in the cable? b) What is percentage elongation?	a) Original length = 3 m Stretched length = 3.2-3 = 0.2m $\varepsilon = \frac{\Delta l}{l} = \frac{0.2}{3} = 0.067$ Strain is 0.067 b) Percentage elongation if the question asked $0.067 \times 100\% = 6.7\%$
Young's modulus; $Y = \frac{\sigma}{\varepsilon}$	It is called also the elastic modulus and it measures the resistance of a solid to a change in its length	
Graphs of stress and straiin		For small stresses the object will return to its initial length when the force is removed. The **elastic limit** of a substance is defined as the max stress that can be applied before it becomes permanently deformed and does not return to its initial length. As the stress increases the curve is no longer a straight line and the material ultimately breaks at the **breaking point**.
More definitions from graphs	A student tests four materials and plots their stress versus strain characteristics on the same set of axes. Her results are shown on the right. An 'x' indicates the fracture point of the materials.	.

		
Strongest	1-Which material is the strongest? Justify your answer	The strength of a material is the largest stress that it achieves up to fracture. Material A withstands the largest stress.
Stiffest	2- Which material is the stiffest? Justify your answer.	Young's modulus indicates the stiffness of a material. ***The greater the value of Young's modulus, the stiffer the material***. Young's modulus is the gradient of the stress versus strain characteristic in the linear region. Material A has the greatest value of Young's modulus.
Toughest	3- Which material is the toughest? Justify your answer.	Toughness is indicated by the area under the stress versus strain characteristic up to the point of fracture. This gives the ***total strain energy absorbed by a cubic metre*** of the material before it fractures. Material C has the greatest area under its characteristic.
Britle	Which material is the most brittle? Justify your answer.	Brittle materials experience little or no plastic deformation before fracture — less than about 5% strain. Material A
Strain Energy	Bungee ropes have good absorbing characteristics or otherwise the jumper would be stopped to quickly resulting in injury. Building are constructed in such a way as to be able to absorb large amounts of energy. We use strain energy to describe the additional energy stored in a material when it is deformed. Strain energy is equal to the work done to deform a material	If the material returns to its original shape when fore is removed, the energy that was stored is called elastic strain energy. STRAIN ENERGY PER UNIT VOLUME = can be determined from the area under the stress-strain graph up to that particular strain. Strain energy per unit volume is usually measured in joules per cubic metre (Jm^{-3})

Problem	The stress-strain characteristic of a particular material are shown on the left The test specimen was 200 mm long and 12 mm in diameter. If the specimen fractured at a strain of 3×10^{-3}, estimate the energy required to fracture it.	
	Calculate how much energy per unit volume is needed to strain the material 0.2% ($=\dfrac{0.2}{100}=2\times10^{-3}$)	The energy per unit volume is found from the area under the curve up to a strain of 2×10^{-3} Tricky question notice that the strain is given in percentages so you will need to change it into proper format So 02% is $=\dfrac{0.2}{100}=2\times10^{-3}$ $A=\dfrac{1}{2}\times(2\times10^{-3})\times(240\times10^{6})$ $A=2.4\times10^{5}\ \mathrm{Jm^{-3}}$
	The test specimen was 200 mm long and 12 mm in diameter. If the specimen fractured at a strain of 3×10^{-3}, estimate the energy required to fracture it.	Now we need to find the area under the graph up to the fracture point. We can use the area from before and then work out the area between the dashed lines. The area between the dased lines consists of a rectangle and a triangle Area of rectangle $=\dfrac{0.1}{100}\times240\times10^{6}=240000$ Area of triangle above $\dfrac{1}{2}\times(280-240)\times10^{6}\times(\dfrac{0.3}{100}-\dfrac{0.2}{100})=20000$ Area between dased lines is 240000+40000= 260000 Add the other area from 0 to 240 MPa Total area = 260000+240000 = 500000 Now that is the energy per volume. So we need to find the volume of the shape which is a cylinder Volume of cylinder - $\pi r^{2}h=3.14\times(0.006)^{2}\times0.2=2.2608\times10^{-5}$ Multiply this by 500000 gives us 11.3 J

Elastic Behaviour	Material returns back to original length when the stress is removed	In high winds the top of the Rialto Tower can move as much as 250mm. When the wind stops the building recovers its vertical position. if it did not it would develop a permanent lean which in time would be castrosphic Important that structures like buildings and bridges return to their original shape when the load is removed. Deformation that disappears when the stress is removed is called elastic deformation.
Plastic behaviour	Deformation that does not completely recover once the load is removed is called plastic deformation.	Example of a piece of chewing gum. When you stretch it, the deformation is not reversible. The stress applied to different materials **does not always cause an instantaneous deformation** in the material.
	MATERIALS	
Materials	When we add different elements to materials scientists have discovered that this alters their properties	Silicon added to aluminium makes aluminium stronger Adding nickel, chromium to steel makes it more resistant to corrosion.
Metals	People have used metals to create different shpaes by rolling, forging or drawing them out.	Examples: Gold- soft metal used for jewellery Lead= once used for irregular gaps around chimneys
	Steel	Strong in compression and tension and a vital component in reinforced concrete. There are many different types of steel but they are all iron based alloys. In general increasing the percentage of carbon maes steel stronger and less ductile. The transition from elastic to plastic behaviour in steel usually occurs suddenly and then breakage occurs.
	Ceramics	Ceramics are hard resistant to corrosion and can take high temperatures. Many are familiar with floor and wall tiles , toilet pans etc. Ceramics are also used to hold structures up. They are brittle and not strong in tension but strong in compression.
	Polymers	Polymers show a wide range of properties. While rubber is very elastic , polystyrene is brittle and behaves plastically.
	Composites- Made by combining two or more different materials to create a single material with enhanced characteristics. Examples timber, concrete etc	Concrete can be considered to be an artificial rock made by mixing cement with sand, rock and water. The concrete has different properties to rock and the cement paste. It is a brittle material and is used for its compressive properties
		Reinforced and prestressed concrete_ Tensile forces do occur in concrete, so to resist cracking other materials are sometimes introduced into concrete. The most common addition is steel reinforcing bars or mesh placed on the tension side of the concrete. Prestressed concrete is often used in bridge construction where large spans are involved. High strength steel tension wires are placed through ducts laid inside the concrete. Before the structure is commissioned these wires are

		stretched and anchored in place. The tensile force in the wires is resisted by the concrete and therefore places the concrete in compression. So when the beam is loaded the tension in the steel increases but the concrete remains in compression.

Balancing forces and Turning forces

For a structure to remain standing two conditions must be met:

Condition 1- $\sum F = 0$

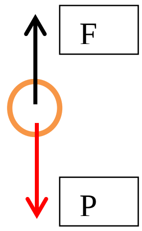
Condition 2- $\sum \mathrm{T} = 0$

Let us explain these two conditions

Condition 1- $\sum F = 0$

What this means that all the forces acting on that body must result in a net force of 0 N. If the net force is not zero then obviously the body will move and it will not be in equilibrium.

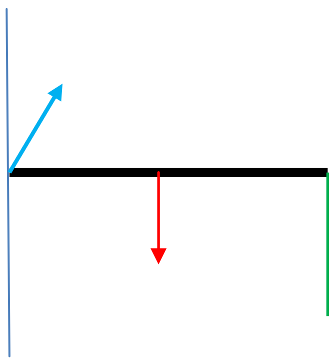
Example



If these two forces, F and P are equal to each other then the body will not move up or down. We say that the body sum of forces is equal to zero.

But what happens if the body is attached to a wall at a particular point?

For example



At the point where the board (black line) connects to the wall there is a reaction force (blue line), but we are not sure which way that reaction force is acting. The green line is the weight of the person which is acting downwards, and the red line is the weight of the board (which we can imagine is acting through the centre of the boards length.

In these cases we need to take the components of the blue force and break them into a horizontal and a vertical component and then we can apply the condition 1, namely that the net forces acting should add up to zero if the body is to be in equilibrium.

How in the case above there is always a turning force to consider which brings us to the next condition

Condition 2- $\sum T = 0$

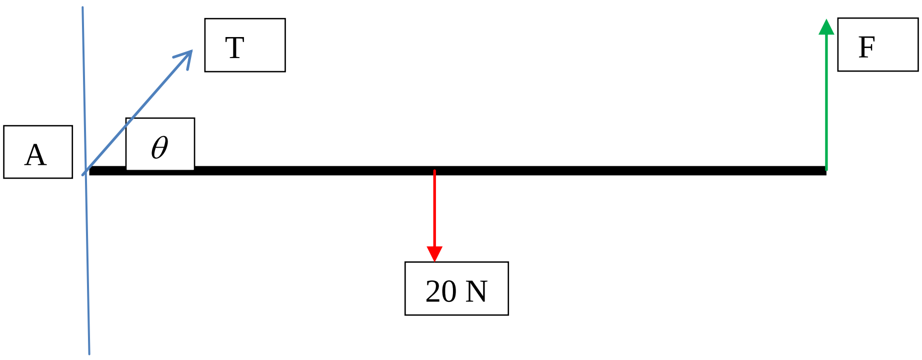
Now to understand how this works we need to define what is this turning force, which we all torque

Turning force = $F_{\perp} \times d$

What does this mean?

To find the turning force we multiply the force by the distance. However we must only multiply the perpendicular component of the force with the distance.

Let look at a simple example;
The board is 8 m long and let's say that we can imagine that its centre of mass where we can imagine all the weight of the board is located in the middle (4 m from point A)



Now we take the torque from point A

If this object is to be in equilibrium then the twisting forces must be equal to zero

So if we take our measurements from point A we can then ignore what is happening at point A and concentrate on the two forces which we know which way they are acting. So by taking our torques or moments (another word) we can eliminate that problem

Force F has a tendency to move the board anticlockwise and the torque is $F \times 8$

Force of the board weight , 20 N, is acting downwards and it will move the board in a clockwise direction, so its torque would then be $20 \times 4 = 80$

So if these two torques are equal to each other then the sum of the torques would be zero and there would be no twisting movements

Anticlockwise torque = Clockwise torques

$$80 = F \times 8$$

$$F = \frac{80}{8} = 10$$

So the force acting upwards is equal to 10 N

Now if we wanted to find the forces at point A, it would involve a little bit of mathematics

We would use the condition 1, which basically means that the sum of all the forces will be zero

Vertically the forces would be the following

$$T \sin\theta + F = 20$$

Horizontally the forces would be the following

$$T \cos\theta = 0$$

Now we know that F = 10 N, so we can substitute that into the first equation

$$T \sin\theta + F = 20$$

$$T \sin\theta + 10 = 20$$

$$T \sin\theta = 10$$

$$T = \frac{10}{\sin\theta}$$

Put that into the other equation and we have

$$T \cos\theta = 0$$

$$\frac{10}{\sin\theta} \times \cos\theta = 0$$

$$10 \cot\theta = 0$$

$$\theta = \cot^{-1}(0)$$

$$\theta = 90$$

So the angle is 90, thus the force is parallel with the board

$$T \sin 90 + F = 20$$

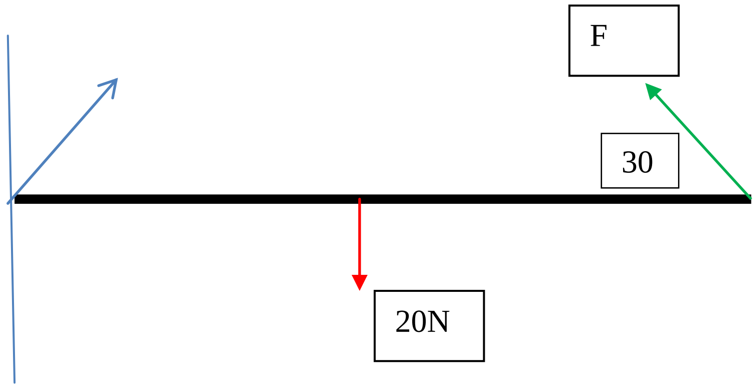
$$T + 10 = 20$$

$$T = 10$$

Which makes perfect sense in this case since the vertical forces will equal the downwards forces

Example 2 –
The question states that the beam is in equilibrium what is the force F now (green force)

A little more challenging. Everything is the same except that the Green force is at and angle of 30 degrees to the horizontal beam, what would the force be then?



Since it is in equilibrium then the torques or moments must be equal to zero

So we take the moments about Point A (where the black beam meets the wall)

Clockwise turning moments- $20 \times 4 = 80$

Anticlockwise turning moments- $F \sin 30 \times 8$

Since these two moments must be equal to each other to be in equilibrium we therefore have the following

$$F \sin 30 \times 8 = 80$$
$$F4 = 80$$
$$F = 20$$

So the force now must be equal to 20 N

More Questions to try

Trusses

In building trusses are used to support ceilings or floors. A struss is a structure comprising of one or more triangular units constructed with straight memebers whose ends are connected at joints referred to as nodes.
External forces and reactions to those forces are considered to act only at nodes and result in forces which are either tension forces or compression forces.

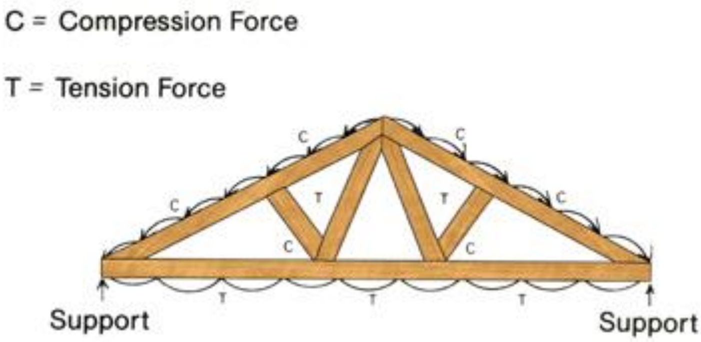
A truss is made up of members that are connected by means of pin joints and which are supported at both ends by means of hinged joints or rollers.

Newton's laws apply to the structure as a whole as well as to each node or joint. In order for any node to remain in equilibrium the following conditions must be true:

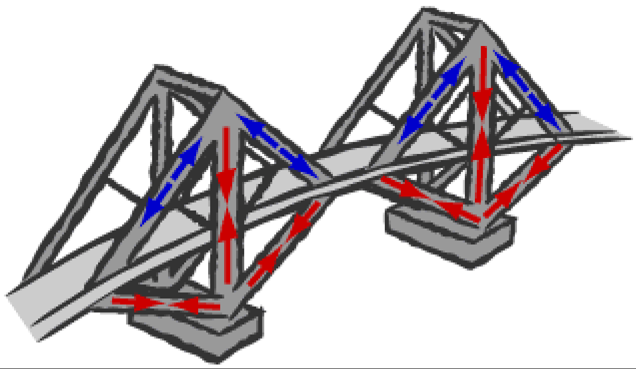
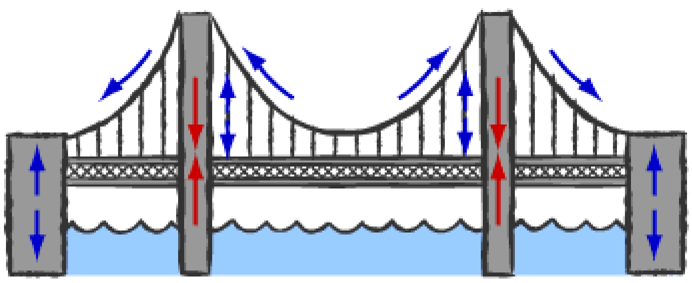
- all horizontal forces + all vertical forces + all moments acting about the node must equal zero.

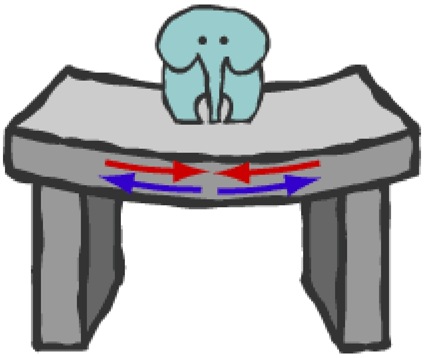
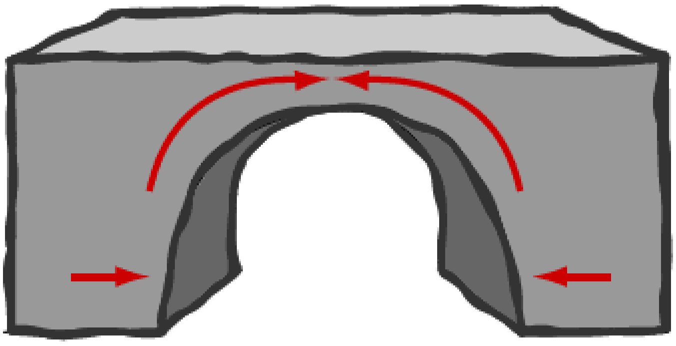
Example

In many common types of trusses it is possible to identify the type of force which is in any particular member without undertaking any calculations. The example below is a common ‘A’ type gable truss with a uniformly distributed load along the top and bottom chords. This is due to the transfer of the load of the tiles through the tile battens and the ceiling load through the ceiling battens.
This means that the chords are subjected to bending forces as well as compression and tension forces. This loading arrangement would result in the top chord restraining compression plus bending forces. The short web is in compression and the long web is in tension. The geometry of both ‘A’ &‘B’ type gable trusses is arranged so that under normal conditions, the longer webs are in tension and the shorter webs in compression. This is done to economise on the size of the timber required for the compression webs.



Bridges

	Truss Bridge: Forces Every bar in this cantilever bridge experiences either a pushing or pulling force. The bars rarely bend. This is why cantilever bridges can span farther than beam bridges.
	Suspension Bridge: Forces In all suspension bridges, the roadway hangs from massive steel cables, which are draped over two towers and secured into solid concrete blocks, called anchorages, on both ends of the bridge. The cars push down on the roadway, but because the roadway is suspended, the cables transfer the load into compression in the two towers. The two towers support most of the bridge's weight.

	Beam Bridge: Forces When something pushes down on the beam, the beam bends. Its top edge is pushed together, and its bottom edge is pulled apart.
	Arch Bridge: Forces The arch is squeezed together, and this squeezing force is carried outward along the curve to the supports at each end. The supports, called abutments, push back on the arch and prevent the ends of the arch from spreading apart.