Projectile Motion

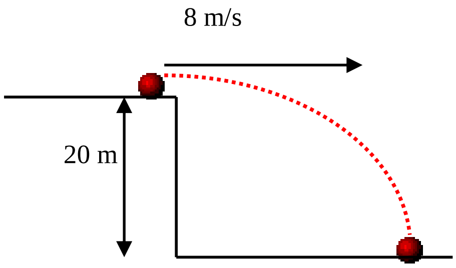
Quick concepts regarding Projectile Motion

Analysing Projectile Motion

	Investigate and analyse the motion of projectiles near the Earth's surface **including a qualitative description of the effect of air resistance**;

Let us break projectile motion into two cases; Case 1- off a horizontal cliff face and Case 2 launched off the ground at an angle

Case 1: Horizontal Projectile Motion



Once the ball leaves the cliff the only force that will act on it is the force of gravity.

Normally we break the motion into two components: the horizontal component and the vertical component

Horizontal Component

Since no force acts in the horizontal direction this velocity will remain the same, namely 8m/s in the horizontal direction

$$v = u + at$$

$$v_h = u_h + at$$

since there is no acceleration in the horizontal direction we have

$$v_h = u_h = 8ms^{-1}$$

Vertical Component

Once it leaves the cliff it begins to experience the force of gravity and its starts to have a vertical velocity downwards

We can work out the value of the vertical velocity simply by using the equations of constant acceleration

$$v = u + at$$

Now we can add a subscript to show that we are looking at the component in the vertical direction

$$v_v = u_v + at$$

So calling down as positive we can find the velocity in the vertical direction at any given time, as it will be given by the equation above

But we will need to know how long it will be in the air before it hits the ground. So we use the equation of constant acceleration

$$s = ut + \frac{1}{2}at^2$$

$$s_v = u_v t + \frac{1}{2}at^2$$

Notice we have used the expression s_v which means the displacement in the vertical direction.

In the problem above the vertical displacement is 20metres and since it just leaves the cliff the initial vertical velocity, u_v is initially 0

There we get the following equation

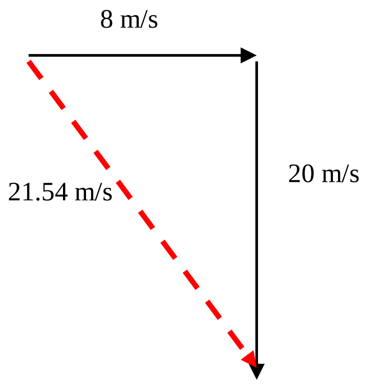
$$s_v = u_v t + \frac{1}{2}at^2$$
$$20 = \frac{1}{2}at^2$$
$$20 = \frac{1}{2} \times 10 \times t^2$$
$$20 = 5t^2$$
$$t^2 = \frac{20}{5}$$
$$t^2 = 4$$
$$t = 2$$

Now we know how long the ball stays in the air, namely 2 seconds so we can find the final vertical velocity just before it hits the ground

$$v_v = u_v + at$$
$$v_v = 0 + 10 \times 2$$
$$v_v = 20ms^{-1} \text{ downwards}$$

Now what is the velocity on impact with the ground?

Since velocity is a vector we need to add the two velocities; the horizontal velocity and the vertical velocity



So using Pythagoras we can find the velocity on impact

$$c^2 = a^2 + b^2$$
$$c^2 = 8^2 + 20^2$$
$$c^2 = 64 + 400$$
$$c^2 = 464$$
$$c = 21.54ms^{-1}$$

But what about how far it goes in the horizontal direction, this is called the range

To find the range we simply multiply the horizontal vertical with the time it is in the air

Now if we would use the equation of motion we would get the following equation

$$s = ut + \frac{1}{2}at^2$$

$$s_h = u_h t + \frac{1}{2}at^2$$

Since there is no force in the horizontal direction , therefore there is no acceleration in the horizontal direction our equation becomes the following

$$s_h = u_h t$$

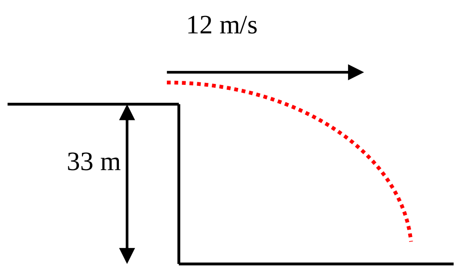
$$s_h = 8 \times 2$$

$$s_h = 16m$$

Summary for doing problems involving case 1

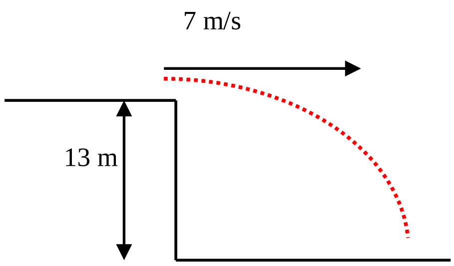
Horizontal component	**Vertical Component**
velocity does not change-U_h	Vertical velocity does change -U_v
Range= $u_h \times t$	To find the time we need to use the equations of constant acceleration and make sure we pick down as positive $s_v = u_v t + \frac{1}{2}at^2$
	To find the final velocity we need to add vectorially the horizontal component with the vertical component

Try a few questions now on this type of projectile motion



Find the following:

a) Find the time it takes to reach the ground
b) Find the horizontal velocity at t = 0.3 sec
c) Find the vertical velocity at t = 0.3 sec
d) Find the resultant velocity at t = 0.3 sec
e) Find the resultant velocity just before it hits ground.



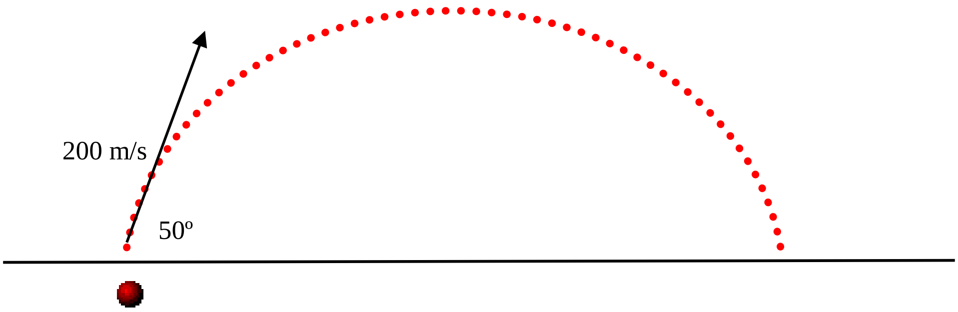
Find the following:

f) Find the time it takes to reach the ground
g) Find the horizontal velocity at t = 0.3 sec
h) Find the vertical velocity at t = 0.3 sec
i) Find the resultant velocity at t = 0.3 sec
j) Find the resultant velocity just before it hits ground.

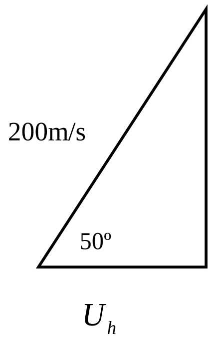
Case 2

Case 2 is a little more involved

Look at the diagram below



Once again we break the problem into two components the horizontal component and the vertical component

<table>
<tr><th>Initial Horizontal Component</th><th>Initial Vertical Component</th></tr>
<tr><td>


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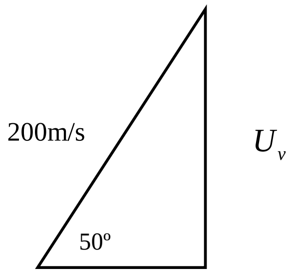


</td></tr>
<tr><td>$U_h = 200\cos 50$</td><td>$U_v = 200\sin 50$</td></tr>
</table>

Now that we have broken the problem into the two components we take each component one at a time and find the relevant information

Horizontal Component

- The initial horizontal component will not change provided the missile in on the same level (most of the time)
- No force acting in the horizontal direction, therefore no acceleration in the horizontal direction

Vertical Component

- This component changes as there is a force acting in the vertical direction, the force of gravity
- If we pick up to be positive then the acceleration will be negative
- At the highest point vertically there will be no more vertical velocity (except for the horizontal velocity), so we can use the equation of constant acceleration to solve this problem
- When the object returns to the ground, the vertical displacement will be zero

Lets us therefore analyse the question to see what we can learn

Question 1: What is the initial vertical velocity of the missile?

Answer:
Since we have already done it from the previous page the initial vertical velocity
$U_v = 200\sin 50 = 153.21ms^{-1}$

Question 2: What is the initial horizontal l velocity of the missile?

Answer

$U_h = 200\cos 50 = 128.56 ms^{-1}$

No remember this horizontal velocity will remain constant for the entire question provided it stays on the same horizontal level

Question 3:What is the vertical velocity after 1 second?

Answer

Since we are asked to find the vertical velocity we will need to use vertical components. Remember we have chosen to use up as being positive so we need to adjust the equations accordingly

$$v_v = u_v + at$$
$$v_v = 153.21 - 10 \times 1$$
$$v_v = 143.21$$

Question 4: What is the horizontal velocity after 1 second?

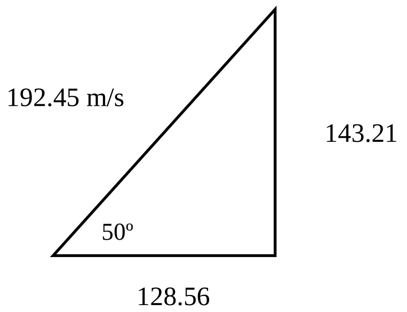
Answer

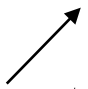
The horizontal velocity does not change therefore it is 128.56 m/s

Question 5: What is the velocity after 1 second?

Answer

This is a tricky question since it is asking us to find the velocity after 1 second, meaning we will need to add the two vectors, i.e the horizontal velocity and the vertical velocity and then using Pythagoras find the resultant velocity.





The answer is 192.45 m/s in the direction given by the two vectors

Question 6: How high will the missile go?

Answer

Once again we are dealing with vertical components, so we will need to use vertical velocities and let us not forget the initial direction-(which is upwards as being positive)

At the top of the height, the vertical velocity will be zero, so we can use the following equation

$$v^2 = u^2 + 2as$$

$$v_v^{\ 2} = u_v^{\ 2} + 2as_v$$

$$\begin{aligned}
&v_v^{\ 2} = u_v^{\ 2} + 2as_v \\
&0 = 153.21^2 + 2 \times 10 \times s_v \\
&0 = 153.21^2 + 20s_v \\
&20s_v = -23473.3 \\
&s_v = 1173.66
\end{aligned}$$

Question 7: How long does it take to get to the maximum height?

We can use many different approaches but let us use a very simple method. At the top of the objects height, the vertical velocity will be 0 and we can then use one of the simple equations to find the time it takes

$$\begin{aligned}
&v = u + at \\
&v_v = u_v + at \\
&0 = 153.21 - 10t \\
&t = \frac{153.21}{10} = 15.321
\end{aligned}$$

So it takes just a bit over 15 seconds or to be precise 15.321 seconds.

Question 8: How long is the missile in the air?
Answer

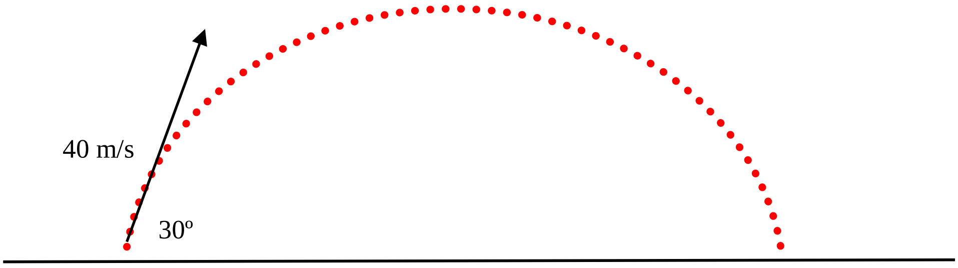
Ignoring air resistance, and looking at the symmetry of the path the answer is twice the time to get to the highest point. so it will be 30.642 seconds in the air.

We could have used another method but this is one of the simplest ways of solving it.

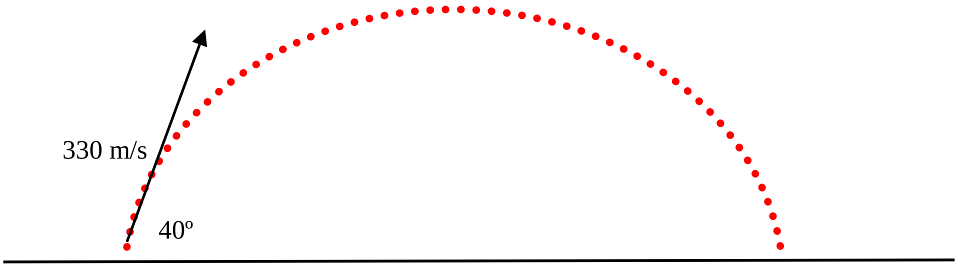
Question 9: Find the final velocity it hits the ground with?
Answer
Ignoring air resistance it should hit the ground with the initial velocity it was projected i.e 200m/s

Another way
Find the final vertical velocity using $v_v = u_v + at$ and then add the horizontal velocity with this to find the final velocity

Questions to try now



a) How far will this object stay in the air for?
b) How far will it go? (its range)
c) Find the initial vertical velocity of the object?
d) Find the initial horizontal velocity of the object?
e) How long does it take to get to the maximum height?
f) What is its maximum height?
g) What is the kinetic energy at the maximum height?
h) What is the resultant velocity just before it hits the ground?
i) After 3 seconds how high is the object vertically?
    a. What is the objects horizontal distance from the start at 3 seconds?



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