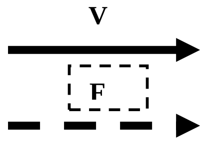
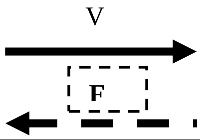
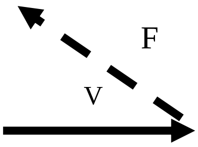
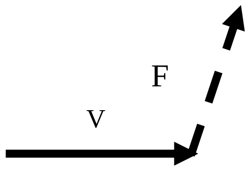
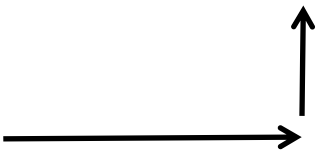
Circular Motion and Gravity

Brief Notes for Unit 3 VCE Physics

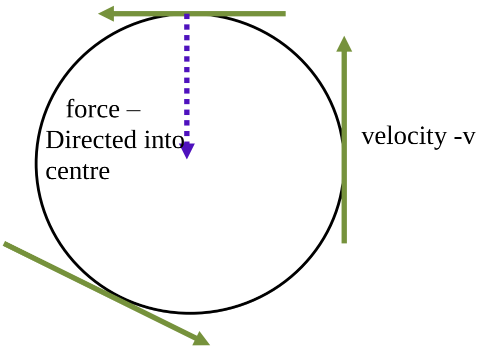
2/20/2010

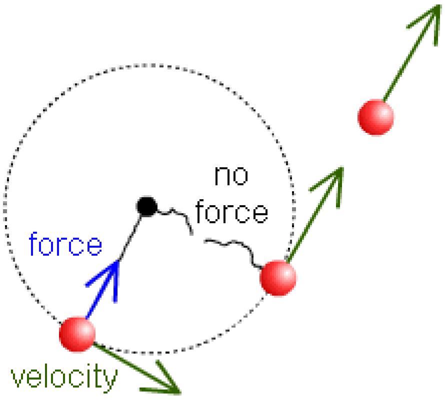
Circular Motion

Some general facts about forces before we begin

Forces	What happens
	If force-F- acts in the direction of motion , the object will speed up
	The force will cause the object to slow down in this case but there will not be a change in direction
	The force will cause the object to slow down as there is a component of the force in the horizontal direction.
	The force will cause the object to increase as there is a component of the force in the horizontal direction
	The force is at right angles to the direction, what will happen here is that there will be a change in velocity and therefore an acceleration or force will occur No change in speed but a change in direction therefore a force will exist. This is exactly what happens in circular motion.

Case 1- Horizontal Circular Motion-motion in a level plane

	When a body travels in a circle, the force is always towards the centre, while the velocity is tangential. The force is always at right angles to the velocity. The force is usually provided by a cord.

	If the cord breaks the force is removed and the circular motion ceases. The object will fly off in the direction it was travelling at the time of the break. If this was originally an object being swung in a vertical circle rather than a horizontal one, the object would move off into projectile motion. You would have to analyse the vertical and horizontal components of the motion separately from the point where the cord broke.

Formulae

$v = \frac{2\pi r}{T}$	Velocity- v Radius of circle- r Period = T = time for one complete rotation, time it takes to go around the circle once
$T = \frac{1}{f}$	Period- T-time for one rotation Frequency-f-number of rotations per seconds
$a = \frac{v^2}{r}$	Acceleration-a Velocity-v Radius-r
$F = \frac{mv^2}{r}$	Force-f Acceleration-a Velocity-v Radius-r

Discussion about the force
In order for an object to undergo circular motion, a force must act. Picture an object that has some velocity. What will happen to it if no forces act on it? Well, according to the first law, it will continue to move with a constant velocity. It will follow a straight-line path.

To make it change direction a force must act on it. In order to make it changes direction constantly, a force must act on it constantly.

What is the direction of the force needed to do this? Well, when you spin something in a circle, what do you have to do? You just pull it towards the center as you go around and around.

The object gets accelerated towards the center. We call this the ***centripetal acceleration***. The equation for the centripetal acceleration is:

$$a_c = \frac{v^2}{r}$$

$\boldsymbol{a_c}$ is the centripetal acceleration, $\boldsymbol{v}$ is the linear or tangential speed, and $\boldsymbol{r}$ is the radius of the circular path.

- A rotating object has a linear speed of 1.5 m/s. It undergoes a centripetal acceleration of 3.6 m/s^2. What is the radius of the mass's circular motion?

$$a_c = \frac{v^2}{r} \qquad r = \frac{v^2}{a_c} \qquad r = \left(1.5\frac{m}{s}\right)^2 \left(\frac{1}{3.6\frac{m}{s^2}}\right) = \boxed{0.62\, m}$$

The force that brings about this acceleration is called the ***centripetal force***. Its direction is also towards the center of the circular path. Centripetal means "center seeking". The centripetal force changes the direction of the object's velocity vector. Without it, there would be no circular path. **The centripetal force is merely a convenient name for the net force that is towards the center. It is always caused by something – it could be caused by the force of gravity, the reaction force between the control surfaces of an airplane with the air, &tc.**

When you rotate a ball around your head in a circle, the centripetal force is supplied by the tension in the string.

Centrifugal force: You may have heard of the centripetal force before you studied physics. It is possible. Most people don't use the term however. Instead they talk about the ***centrifugal force***. Just what the heck is that?

Okay, you've seen the word "centrifugal force", now forget it! Here's the deal. The centrifugal force is the thing that people blame for the feeling that things seem to be pushed away from the center of spin during rotation. You place a coin on a turntable and then spin it really fast. What happens to the coin? In your mind you picture the coin flying straight away from the center of the record. The individual who had not studied physics would say this was because of the centrifugal force.

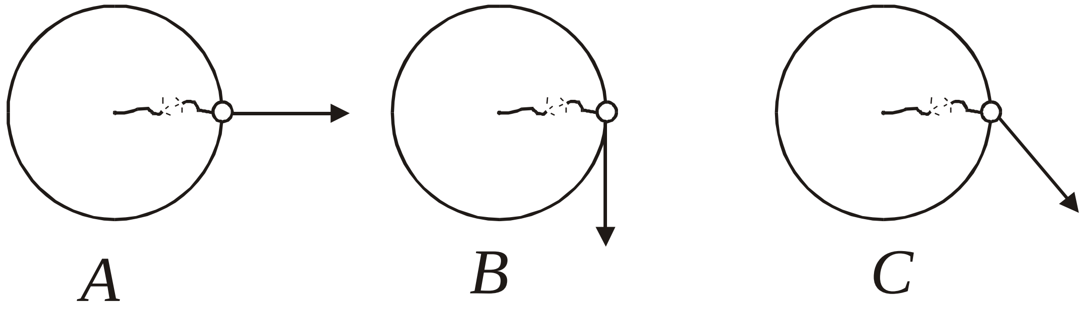
Here is an important concept:

> ***The centrifugal force is a fictional force. There is no actual force that is pushing away from the center of a rotating system.***

You feel this "centripetal force" when you are a passenger in a car that makes a turn. When the car enters the turn, you feel as if you have been pushed into the door, away from the center of the circular path the car is making. So you think, "Hey, I'm being pushed into the door so there must be a force pushing me away from the center of the turn."

That's certainly how you feel at any rate.

Remember the problem at the beginning of this topic? About the path a ball would follow if the string were to break? You had you three of your basic choices:



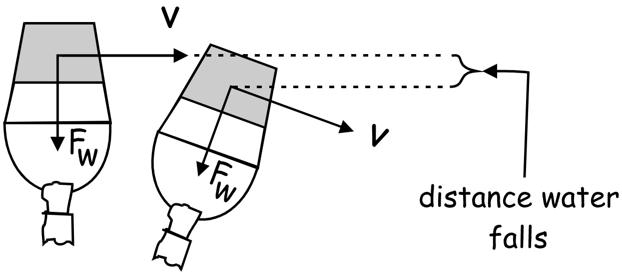
The correct choice, you've hopefully (actually, the Physics Teacher should say "it is to be hoped", but that sounds very pompous, so we won't say that) figured out that the correct path is ***B***. Why? Well at the point in the circle where the string breaks, the ball has a velocity that is tangent to the circular path. The string is providing the centripetal force – pulling the ball towards the center. The ball wants to follow the tangential path because of the first law, but the string won't let it. The string, via the tension it exerts, pulls the ball towards the center, changing the direction of its motion and making it follow the circular path. When the string breaks there is no longer any force to change its direction, so the ball travels in a straight-line path that is tangent to the circle as in the ***B*** drawing in accordance with the first law of motion.

This is sort of what is going on in the car with the passenger.

The passenger wants to travel in a straight-line path at a constant speed in accordance with the ***law***, the first law to be exact. The car however has different ideas. It decides to go in a circular path. It's the tires pushing it toward the center exerting a force to make it all happen. So the car changes direction, but you, the passenger, do not. No force is acting on you. So you go forward in your original direction until you push into the door, which then pushes you toward the center and you then go in a circle as well. The third law rears it head – you push into the door and the door pushes into you. You feel like you are being pushed into the door, even though there is no real force doing this. It's just the reaction force to you pushing into the door. This is the so called centrifugal force, this sensation of being pushed away from the center.

If the centrifugal force was real, i.e., there ***was*** a force pushing you away from the center of the circular path, then if the door were to suddenly pop open you would fly straight away from the center of the circular path. But of course that does not happen. You would travel in a tangential path. The centripetal force is real, the centrifugal force is not.

Centripetal Force and Gravity: The Physics Teacher did a silly demonstration involving a bucket of water that was spinning in a vertical circle. The water stayed in the bucket and did not fall out.



So what was the deal? Does spinning something in a vertical circle somehow cancel out gravity?

Well, no, gravity is a force that cannot be stopped or canceled. It is always there, anytime you have the appropriate masses.

The water does fall, it falls but the bucket falls with it and catches it.

This only works if the bucket is moving fast enough to catch the water. If the bucket is too slow, then the water ***will*** fall out of it.

The minimum linear speed for this is called the ***critical velocity***.

Critical velocity ≡ minimum velocity for an object to travel in vertical circle and maintain its circular path against the force of gravity.

The same thing is needed for satellites in orbit around the earth or planets in orbit around the sun. They too must travel at the critical velocity.

The critical velocity formula is very simple to figure out.

You just set the centripetal force equal to the weight of the object that is in circular motion. If the two forces are equal, then the object won't be able to "fall out" of the bucket.

$F_C = \frac{mv^2}{r}$ and $F = mg$ Set them equal to each other:

$\cancel{m}g = \frac{\cancel{m}v^2}{r}$ $\quad v^2 = gr$ $\quad v = \sqrt{gr}$

So here is the critical velocity $\quad v = \sqrt{gr}$

- A carnival ride travels in a vertical circle. If the ride has a radius of 4.5 m, what is the critical velocity?

$$v = \sqrt{rg} = \sqrt{4.5\,m\left(9.8\,\frac{m}{s^2}\right)} = \sqrt{44.1\,\frac{m^2}{s^2}} = \boxed{6.6\,\frac{m}{s}}$$

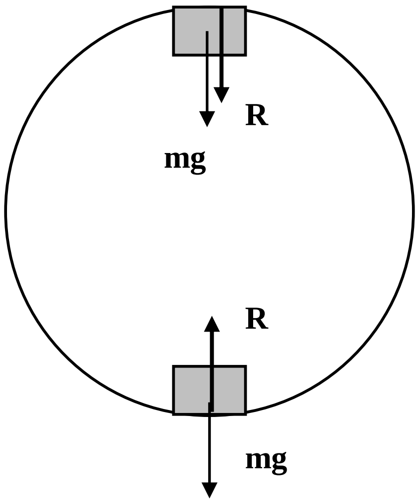
<u>NON-UNIFORM CIRCULAR MOTION</u>

Consider a car passing through a vertical loop. At the top, the force towards the centre of the circle is given by

<u>At the top</u>

$$R + mg = \frac{mv^2}{r}$$

Here the car speed is affected by gravity.
It slows down on the upward section and speeds up on the downhill section.



Except at the top and bottom of the loop, the force of gravity means that there is a component of the car's motion.

At the bottom

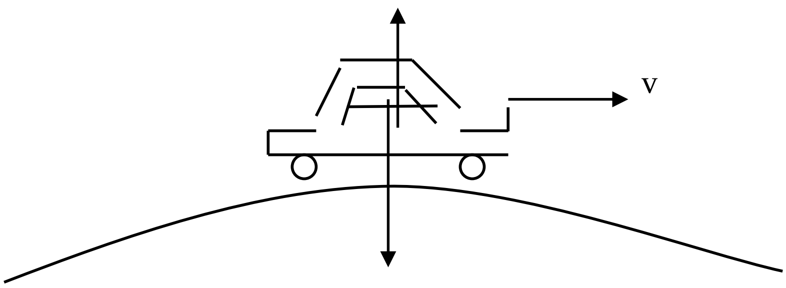
$$R - mg = \frac{mv^2}{r}$$

EXAMPLES OF CIRCULAR MOTION EFFECTS

Person on a swing

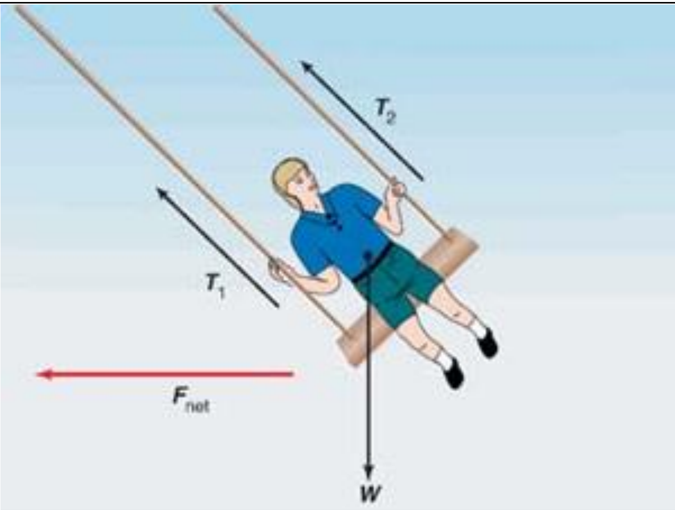
At the bottom of the swing, the forces on the person are the reaction from the seat and the weight force. $R - mg = \frac{mv^2}{r}$. In this case R > mg, so the person 'feels' heavier.

Person in a car going over a hump.

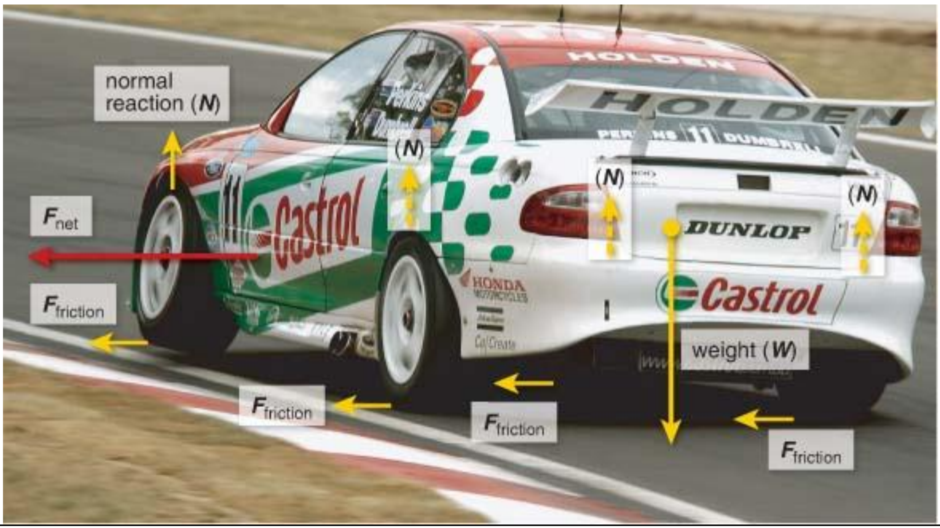


$mg - P = \frac{mv^2}{r}$ $\therefore P = mg - \frac{mv^2}{r}$ ∴The person feels lighter.

A few pictures to see how diagrams are drawn



Tension- Notice how the net force is directed towards the middle of the circle,



Friction

When a car rounds a corner, the sideways frictional forces contribute to the centripetal force. If these frictional forces are not sufficient, the net force on the car will not be towards the centre of the curve.

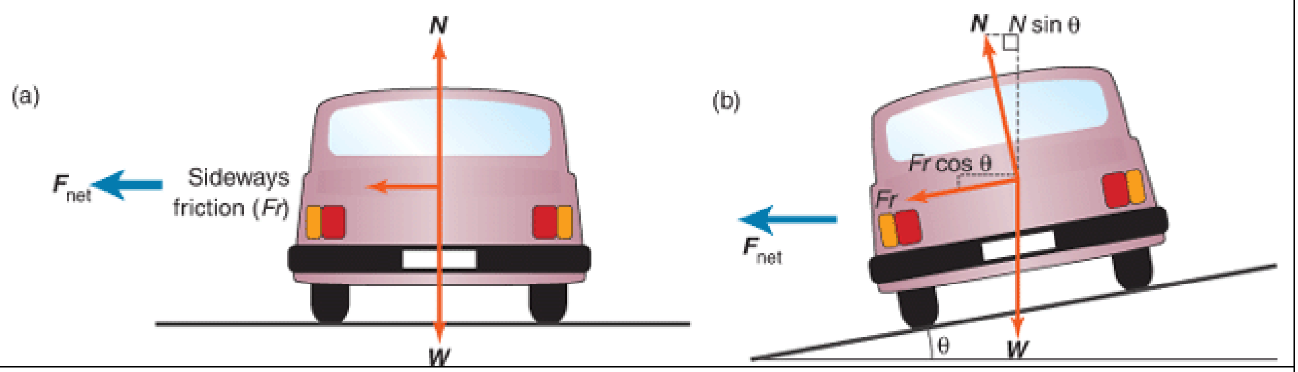
In this situation, the net force is less than the centripetal force required to keep the car moving in a circle and it will not make it around the corner!

The formula for centripetal force shows that as the velocity increases, the force needed to move in a circle greatly increases

($F_{net} \propto v^2$).

This is why it is vital that cars do not attempt to corner while travelling too fast

Leaning- Track athletes, cyclists and motorcyclist s also rely on sideways frictional forces to enable them to move around corners.
To increase the size of the sideways frictional force, which will therefore allow them to corner more quickly, they often lean into the corner.
The lean also means that they are pushing on the surface at an angle, so the reaction force is no longer normal to the ground. It has a component towards the centre of their circular motion.



Banked corners-
The diagram above shows a car travelling around a flat corner (a) and travelling around a corner banked at an angle of θ^o (b)
In the first case – $F_{net} = \text{sideways friction} = F_r = mv^2/r$
For the banked corner – $F_{net} = F_r \cos\theta + N \sin\theta = mv^2/r$
The larger centripetal force means that, for a given curve, banking the road makes a higher speed possible.

Problems on Circular Motion

1	A dog is chain with a rope of length 8 m in the middle of the garden. It takes the dog an average of 12 seconds to complete one lap. a) What is the dog's average speed? b) If the dog has a mass of 50 kg what is its acceleration? c) What is the net force towards the centre of the garden.
2	A car is driven around a roundabout at a constant speed of 8 m/s. The roundabout has a radius of 4 m and the car has a mass of 1200 kg. a) What is the magnitude and the direction of the acceleration of the car? b) What is the magnitude and direction of the force on the car?
3	Find the frequency in seconds if 1 period takes 10 sec
4	

GRAVITY

Main ideas

Concept 1	apply **gravitational** field **and gravitational force** concepts, $g = \frac{GM}{r^2}$, $F = \frac{GM_1M_2}{r^2}$
Concept 2	**model** satellite motion (**artificial, moon, planet**) as uniform circular orbital motion ($a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$)
Concept 3	apply the concepts of weight (*W=mg*), apparent weight (reaction force, *N*), weightlessness (*W=0*) and apparent weightlessness (*N=0*);

Explanation

What is the law of universal gravitation?	Newton discovered that there is a force of attraction between two objects throughout the universe. This force of attraction is determined by the masses of the two bodies and the distance between the two masses

	Here is the equation $F = \frac{GM_1M_2}{r^2}$
What is the Universal constant?	6.67×10^{-11}
	$F = \frac{GM_1M_2}{R^2}$ **G** is equal to 6.67×10^{-11} and M_1 and M_2 is the weight of the two bodies **R** is the distance between the two bodies from centre to centre
What type of force is this?	It is a force of attraction that attracts both bodies towards each other.
What is the gravitational field of gravity? Answer- same as gravitational acceleration at a particular point!	It is a measure of the acceleration that a body would experience at a certain distance from the planet in question $g = \frac{GM}{r^2}$, here M is the mass of the planet and r is the distance from the centre of the planet For example if we would put some numbers into this equation we can find the gravitational field on the surface of the Earth. Lets us put a few numbers. The Earth is not perfectly spherical but we will approximate it as being spherical R – radius of Earth- Mean radius of 6371 km M-mass of the Earth- 5.9742×10^{24} kg G- 6.67×10^{-11} Putting it into the equation gives us $g = \frac{GM}{r^2} = \frac{6.67\times10^{-11}\times5.9742\times10^{24}}{(6371\times1000)^2} = 9.8172$ which is not bad considering that the average value of the acceleration of gravity varies from 9.789 at the equator to 9.832 at the poles
What is apparent weightlessness	When an object falls towards the earth its has no reaction force therefore the person feels as though they are weightless.
Weight	$W = mg$
What sort of equations can we deduce if a body is orbiting another body such as the Moon orbiting the Earth	The Moon circles the Earth therefore the net force on the Moon is towards the centre of the Earth. And in fact the force of attraction between the Earth and the Moon is the same So

	$F_g = F_c$ $\frac{GM_{earth}M_{moon}}{r^2} = \frac{M_{moon}v^2}{r}$ $\frac{GM_{earth}}{r^2} = \frac{v^2}{r}$ $v^2 = \frac{GM_{earth}}{r}$ $v = \sqrt{\frac{GM_{earth}}{r}}$ This gives us the velocity of the Moon as it travels around the earth
We can express this formula a little differently also	$F_g = F_c$ $\frac{GM_{earth}M_{moon}}{r^2} = \frac{M_{moon}v^2}{r}$ $\frac{GM_{earth}}{r^2} = \frac{v^2}{r}$ $v^2 = \frac{GM_{earth}}{r}$ $[v]^2 = \frac{GM_{earth}}{r}$ $\left[\frac{2\pi r}{T}\right]^2 = \frac{GM_{earth}}{r}$ $\frac{4\pi^2 r^2}{T^2} = \frac{GM_{earth}}{r}$ $4\pi^2 r^3 = GT^2 M_{earth}$ $M_{earth} = \frac{4\pi^2 r^3}{GT^2}$ Now if we put a few numbers in we will get a few we can work out the Mass of the Earth T = 27.3 days for the Moon to rotate around the Earth and it has a distance of of centre to centre is 384,403 km $M_{earth} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (384403 \times 1000)^3}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2} = 6.04 \times 10^{24}$ kg Which is not bad considering that the Earth Mass is approximately 5.9742×10^{24} kg
Keplers laws	The ratio of $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ is constant for every object revolving around a particular M

Weightless	An object is weightless in a region where the gravitational field strength is zero, $W = mg = 0$
Apparent weightlessness	Occurs when there is no normal reaction. When a spacecraft orbits the earth it is as though it is free falling thus the astronauts feel weightless

***Gravity in Orbit:* Explanation of weightless**

We all know that the astronauts in space orbiting the earth are "weightless". Does this mean that there is no gravity in space? Well, no. Most of our spacecraft are in pretty low orbits. The distance between the astronauts and the earth is not that much greater than when they are on the earth. Their weight is only about ten percent less than it is on earth. So why are they weightless?

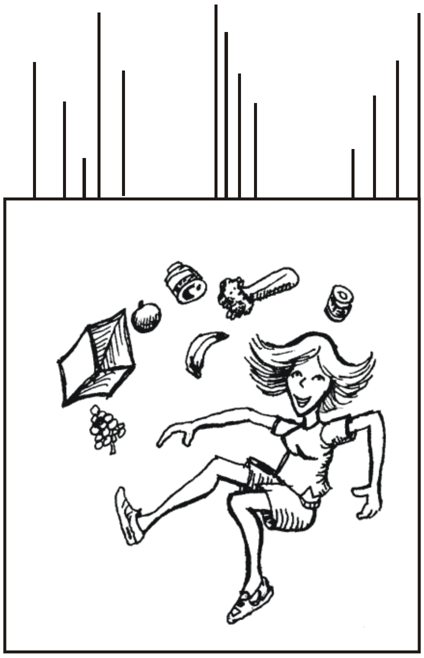
The space shuttle and everything in it are falling towards the earth. It is in a state of freefall. We know that everything falls at the same rate, so everything in the space shuttle is falling at the same speed. Because of this there is no relative motion between the space shuttle and everything in it. There is no sense of up or down and the astronauts no longer feel the force of gravity. It's like being inside an elevator that is falling down the elevator shaft (the cable broke or something). In a normal elevator, one that isn't falling, gravity exerts a downward force on everything. If you stand on a bathroom weight scale, you push down on it and it reads out your weight. But now the cable breaks. You are still standing on the scale, but the elevator, the scale, and you are all falling down accelerating at 9.8 meters per second squared. You no longer exert a force on the scale – it is falling at the same speed that you are. It now reads zero.

Any objects in the elevator would appear to be weightless. If you held a ball outward and then released it, it would not appear to fall down (since it is already falling). It would appear to float in space in front of you. You would think, "Hey, cool, there's no gravity in the elevator. Neat!"

It would be pretty neat too…until the elevator hits the bottom of the elevator shaft.

Okay, let's transfer this idea to the space shuttle. It is, in effect, a falling elevator, one with the advantage of not crashing into the floor – it never hits the earth!

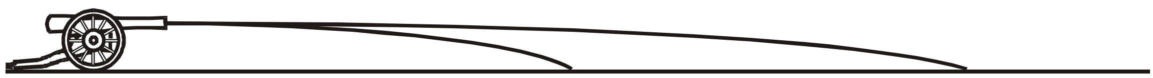
That's why the astronauts are "weightless".



Free Falling in an elevator

Explanation of orbital

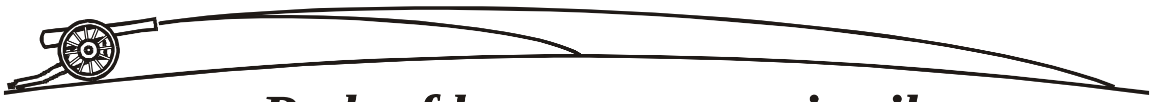
Let’s do us a “mind experiment”. This is an experiment where you think instead of do. Anyway, picture a cannon that is set to fire horizontally. What does the path of the projectile look like?



Path of short range projectile

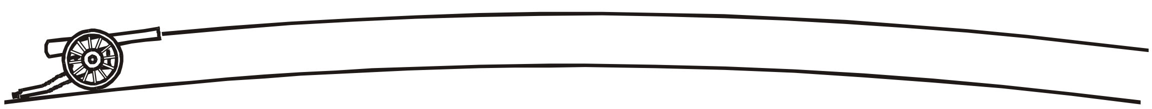
The projectile will follow a curved path. This is because it is being accelerated downwards by the force of gravity. The greater the velocity of the projectile, the farther it will go before it strikes the Earth.

The Earth, however, is not flat, although over short distances we can pretend that it is. So what actually is happening is that the projectile moves over and falls to the ground on a curved surface. So we have a curved path and a curved surface. We have to take the curvature of the Earth into account when firing long range projectiles. Possible paths would look like this:



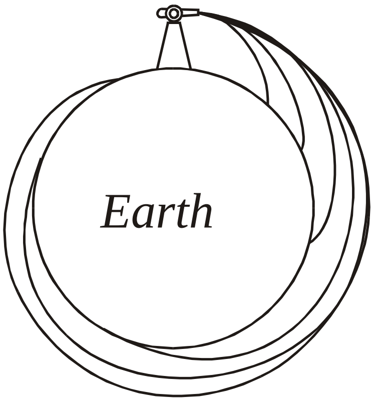
Path of long range projectile

Again, the greater the velocity of the projectile, the greater the range. Newton showed that if the velocity was great enough, the curving path of the falling projectile would match the curved surface of the earth and the falling projectile would never actually hit the Earth. Here is a drawing of this.

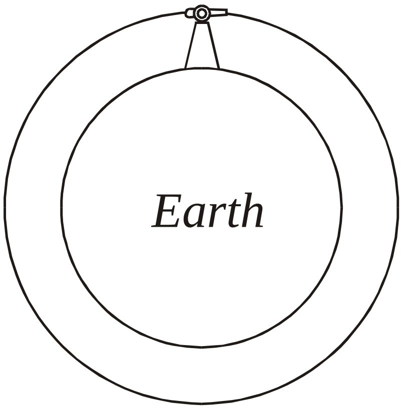


Path of projectile with same curvature as surface of the earth

Newton imagined a mountain on the earth that was so high that its summit was outside of the earth’s atmosphere (this eliminates friction with air). On top of the mountain is a powerful cannon. The cannon fires a projectile horizontally. The projectile follows a curved path and eventually hits the earth. Now we add more gunpowder to the charge and fire another cannonball. This cannonball will travel a greater distance before it too hit the surface of the earth. We keep firing the gun with a bigger and bigger charge. The cannonball goes further and further before it strikes the earth. Eventually the velocity is great enough so that the curved path of the projectile matches the curved surface of the earth and the cannonball never gets closer to the planet’s surface. It keeps falling forever.



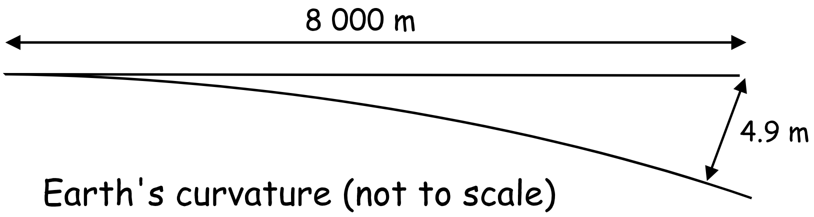
Path of projectile fired with larger and larger charges



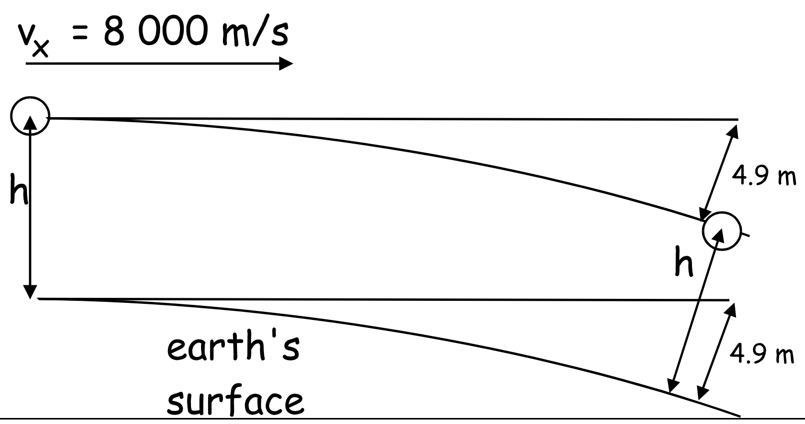
With just the right velocity, the projectile never reaches the earth's surface

That basically is your orbit.

The earth is not flat. It is a sphere and its surface has a fairly constant curvature. The surface drops 4.9 m in 8 000 m of horizontal travel.



If we launch a cannonball with a velocity of 8 000 m/s, it will fall a distance of 4.9 m and travel a horizontal distance of 8 000 m in one second. This means that it will stay at the same height above the earth's surface throughout its path. Of course, if we did this near the surface, we'd have the air slowing the projectile down. We'd also have to worry about the cannonball running into houses and mountains and trees and so forth. Above the atmosphere, however, all these impediments are eliminated.

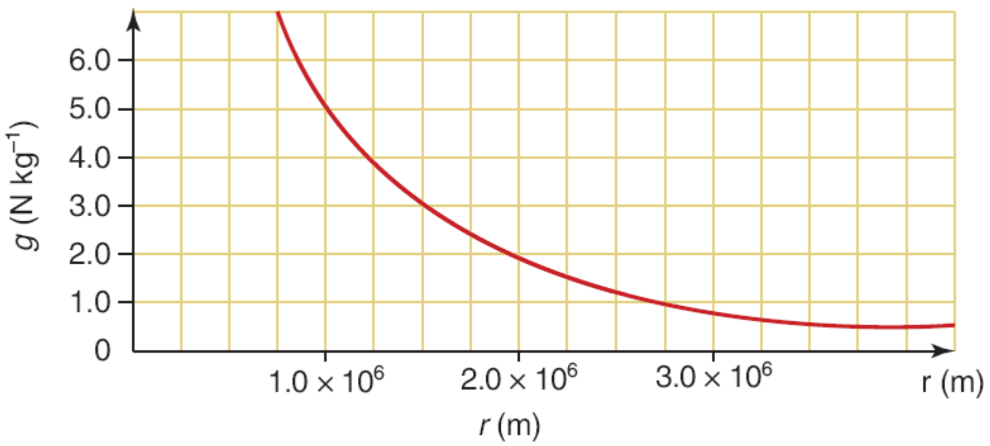


The orbit of the everyday celestial object is described by a combination of the law of gravity, Newton's three laws, and the stuff we just learned about, circular motion.

Problems

Launching satellites

Rockets must do work against gravity to place a satellite into orbit.
Remember that work is the area under a force–distance graph, so the amount of work done against (or by) gravity calculated in this way.



A 100 kg meteorite travels towards the surface of a planet in orbit around a distant star. The graph below shows how the gravitational field strength g changes with the distance r from the centre of the planet.

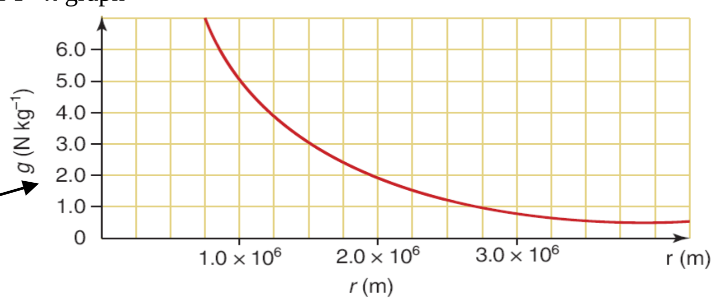
a) What is the weight of the meteorite when it is 1.0×10^6 m from the planet's centre?

a) $W = mg = 100 \times 5 = \underline{500\ N}$

b) How much work is done by the planet's gravity while the meteorite falls from $r = 3.0 \times 10^6$ m to $r = 1.75 \times 10^6$ m?

b) Work = area under F–x graph

Multiply by m to change to a F – x graph



$\Rightarrow$ work $\approx 1.25 \times 10^6 \times 100 + 0.5(1.25 \times 10^6) \approx \underline{1.26 \times 10^8\ J}$

c)	If the speed of the meteorite is $1800\ m\ s^{-1}$ when $r = 3.0 \times 10^6$ m, estimate its speed at $r = 1.75\times 10^6$ m.
c)	when $r = 3.0 \times 10^6$ $E_k = 0.5 \times 100 \times (1800)^2 = 1.62 \times 10^8 J$ when $r = 1.75 \times 10^6$ $E_k = 1.62 \times 10^8 + 1.26 \times 10^{8-} = 2.88 \times 10^8$ $\Rightarrow$ $\frac{1}{2} \times 100\ v^2 = 2.88 \times 10^8$ $\Rightarrow$ $v = \underline{2.4 \times 10^3\ ms^{-1}}$