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Probability Notes

Probability-Part 1

Probability is the mathematics of chance. We are all familiar with the idea of tossing a coin and seeing on which side of the coin it lands on. In this case (assuming it does not land on his narrow rim, there is that possibility no matter how remote) there are only two possibilities- either it shows a head or it shows a tail. Problems involving chance are classified as probability.
What makes probability questions harder than the rest of the questions we have so far seen is the fact we need to read the question very carefully and make sure we make valid assumptions, and there lies the problem with dealing with probability type of questions.

Probability is used in many areas but notably: gambling, finance, statistics, science etc.

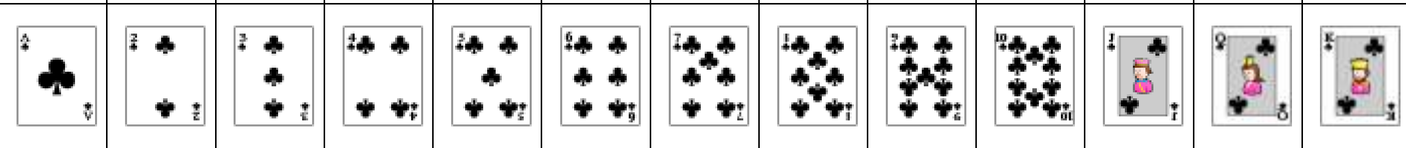
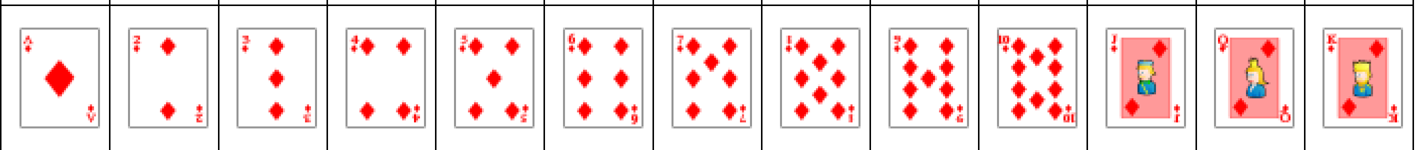
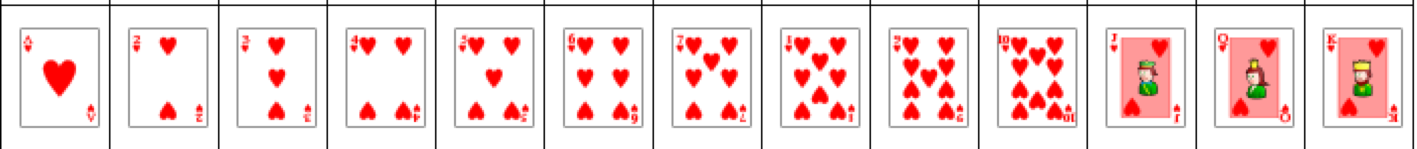
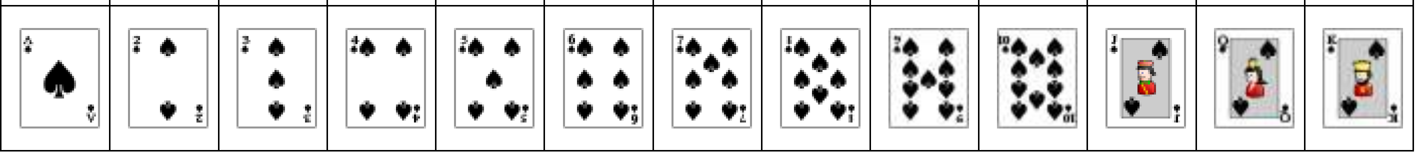
In our study we will look into: discrete probability distributions and also continuous probability distributions

Discrete Probability distributions- deals with events that occur in countable sample spaces. For example throwing dice or picking a certain card from a deck of cards.

Continuous Probability distributions- deals with events that occur in our continuous sample space which incorporate many of the ideas of calculus

Basic Ideas- Preliminary Ideas

A card of deck consists of 4 suits and it made up of a total of 52 cards – (different decks have more cards)

Suit	**Ace**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Jack**	**Queen**	**King**
Club													
Diamonds													
Hearts													
Spades													

Concepts- Words	
Range of Probabilities	All Probabilities lie in the range 0 to 1
Probability of 0	An event that has a probability of 0 is said to be impossible
Probability of 1	An event that has a probability of 1 is said to be certain.
Discrete Variable	One where we can count the successful outcomes and our answers will always be a whole number E.g. the number of heads in a series of coin tosses.
Continuous Variable	Measured on a continuous scale and our answer will always contain decimals E.g. height of students for example 125.4 cm

Lets us consider a very easy example to highlight some of the main ideas of the past Example 1: If a die (single for dice) is rolled once the possible outcomes will be the following: 1, 2, 3, 4, 5 and 6	
SAMPLE SPACE	: the set of all elements under consideration in the question Here in the above example it would be ε = {1, 2, 3, 4, 5, 6}
EVENT	A subset of the sample space, normally shown by a capital letter Let say A is an even number when a die is rolled so the event A= {2, 4, 6} Another event B could be a number greater than 2 occurring, B= {3, 4, 5, 6}
INTERSECTION OF A and B $\rightarrow A\cap B$	$A\cap B$ which means that both A and B occurring Another way of explaining the meaning of $A\cap B$ could be the intersection or overlapping of both set A and set B In the above example $A\cap B$ = {4, 6}
PROBABILITY	PROBABILITY is a numerical measure of the chance of a particular event occurring, for example the chance of getting a 6 from a single throw would be worked out as follows: Pro(a 6 occuring)= $\frac{1}{6}$ Notice there is only 1 six, so that explains the 1 and there are 6 numbers in total, thus 1 in 6 chance
Simple Rules for Probability	$0\le \Pr(A)\le 1$ For all events A in the sample space. (In other words we can only have probabilities occurring with values between 0 to 1 only. $\Pr(A')=1-\Pr(A)$, where A' is called the complement of A $\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$ $\Pr(A\cup B)$ $\rightarrow$ Means probability A or B or both occurring Pr (A) =0 $\rightarrow$ means event A is impossible to happen, no chance Pr (A) =1 $\rightarrow$ event A will occur no doubt, it will happen
INDEPENDENT EVENTS	A and B are independent events if $\Pr(A\cap B)=\Pr(A)\bullet\Pr(B)$
MUTUALLY EXCLUSIVE EVENTS	$\Pr(A\cap B)=0$ $\rightarrow$ That would mean that there was no overlap between events A and B i.e. any elements in common
Example 2	

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Continue from previous example let’s call event A= obtaining a even number {2, 4, 6}

Pr (A is an even number when the die is rolled) = $\frac{3}{6}$

Now let’s call event B, obtaining a number greater than 2 :{ 3, 4, 5, 6}

Pr (B obtaining a number greater than 2 occurring) = $\frac{4}{6}$

Now A and B occurring or overlapping :{ 4, 6}

Therefore $\Pr(A \cap B) = \frac{2}{6}$

Now to work out $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

We substitute the above and we obtain the following:

$\Pr(A \cup B) = \frac{3}{6} + \frac{4}{6} - \frac{2}{6}$
$\rightarrow = \frac{7}{6} - \frac{2}{6}$
$\rightarrow = \frac{5}{6}$
</td></tr>
<tr><td>Example 3</td><td>
Still staying with the same example i.e. that is the die being tossed now lets imagine

Event A= obtaining an even number $\rightarrow$ {2, 4, 6}

Event B= obtaining an odd number $\rightarrow$ {1, 3, 5}

Now A ∩ B = 0 since there is no overlapping between events A and B, $\Pr(A \cap B) = 0$

Pr (A) = $\frac{3}{6}$ and Pr (B) = $\frac{3}{6}$

Now substituting all the above into the equation below we get the following:

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A \cup B) = \frac{3}{6} + \frac{3}{6} - 0$
$\Pr(A \cup B) = \frac{6}{6} = 1$

Probability of 1 means that it must occur and common sense tells us that this would be the case.
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MUTUALLY EXCLUSIVE	When 2 events have no outcomes in common they we call that mutually exclusive $\Pr(A \cap B) = 0$
Definition of Probability	$\Pr(\text{event A occuring}) = \dfrac{\text{number of outcomes in which event A occurs}}{\text{total number of possible outcomes}}$

Example 4

A coin is tossed once. What is the probability of getting a tail?

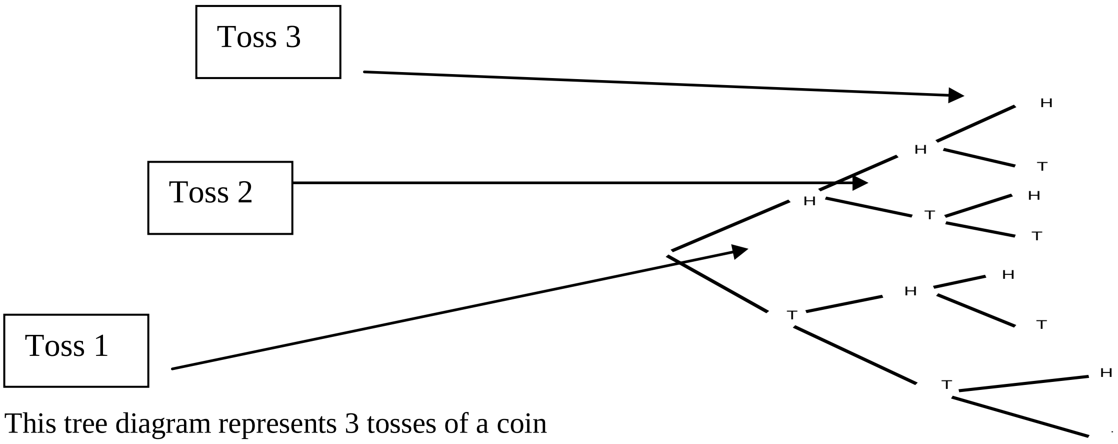
Solution

Event= getting a tail
Total outcomes: only 2 either get a head or a tail

Pr (tail) = $\frac{1}{2}$

Sometimes we use various diagrams to show probabilities or to help us out in working things out.

One of the most common diagrams is called a tree diagram



This tree diagram represents 3 tosses of a coin

Lets call event A = getting a head in each of the three tosses

A= {H, H, H}

Event space= {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

So Pr (HHH) = $\frac{1}{8}$

CAN YOU SEE HOW THE EVENT SPACE CAN BE FOUND FROM THE TREE DIAGRAM?

Probability Tables and Tree Diagrams

Probability tables and Karnaugh maps	**Probability tables**, are also called **Karnaugh maps**, and are useful when only two events are involved.

	A	A'	

B	$\Pr(A \cap B)$	$\Pr(A' \cap B)$	$\Pr(B)$
B'	$\Pr(A \cap B')$	$\Pr(A' \cap B')$	$\Pr(B')$
	$\Pr(A)$	$\Pr(A')$	1

The bottom right-hand cell of the table must be equal to 1as it represents either Pr(A) + Pr(A′) or Pr(B) + Pr(B′); which both represent the complete range of possibilities.

<u>Example 5</u>

The probability that a student in a particular class has fair hair is 0.6 while the probability of blue eyes is 0.7. The probability of having fair hair and blue eyes is 0.5.

Find the probability of having neither fair hair nor blue eyes.

1. Draw up a probability table and include the known values.

	Fair	Not Fair	
Blue	0.5		0.7
Not Blue			
	0.6		1

2. Fill in the remaining values of the boxes and you will get the following

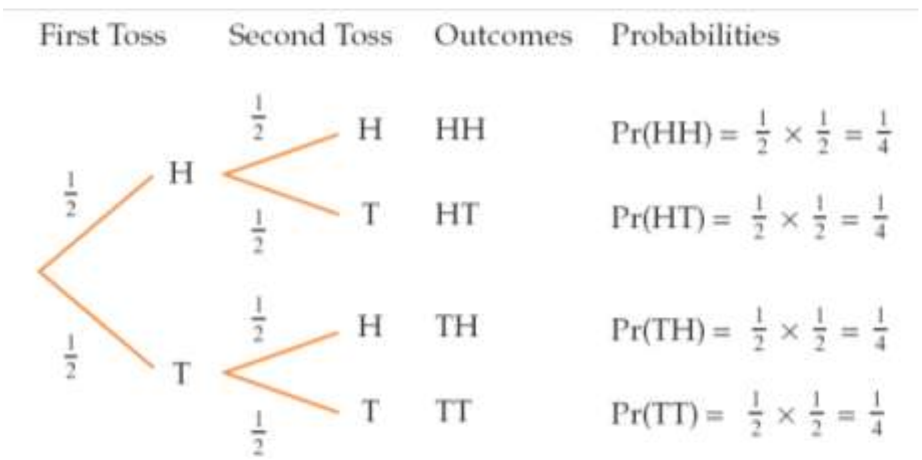
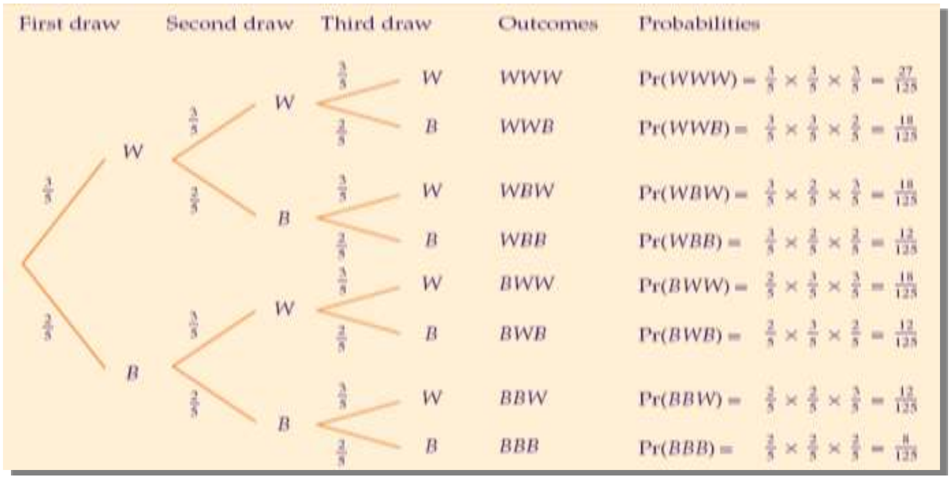
	Fair	Not Fair	
Blue	0.5	0.2	0.7
Not Blue	0.1	0.2	0.3
	0.6	0.4	1

Now we can find the answer to the original question: Pr(neither fair hair nor blue eyes) = 0.2

Tree Diagrams

In most cases a tree diagram is the best way of representing the full range of outcomes, along with their associated probabilities.
Each time there is a branching process the sum of the probabilities of those branches must be 1 and the final probabilities must also equal 1 also.

Example: Tossing a fair coin twice – (same as tossing two coins)
Answer

	
Example 6	A box contains six white cubes and four blue. Cubes are drawn from the box one at a time, and replaced before the next cube is drawn. Find the probability that of the first three cubes drawn exactly two are white. Solution: Read the question carefully- notice that the cubes are replaced meaning that the total number of cubes stays the same. Secondly we have three draws- and we need to find exactly two white and one blue. So we can get WWB or WBW or BWW where W- stands for white cube and B-blue cube It is easier to draw a tree diagram  Now we identify the 'successful' events-WWB or WBW or BWW So we can the probabilities from the tree diagram Pr(exactly 2 white) = $^{18}/_{125} + ^{18}/_{125} + ^{18}/_{25} = ^{54}/_{125}$
Example 7	Find the probability of drawing three hearts in a row from a pack of cards if the cards are not replaced after each draw. Solution Remember a normal playing deck has 52 cards of which 13 are of the suit of hearts. Let us identify the events H - drawing a heart and H' - not drawing a heart

So on **the first draw**

$\Pr(H) = {}^{13}/_{52} = 1/4$

$\Pr(H') = {}^{39}/_{52} = 3/4$

On the **second** draw –

If a **head was drawn first** (12 of the 51) cards left are heads.

$\Pr(H) = {}^{12}/_{51}$

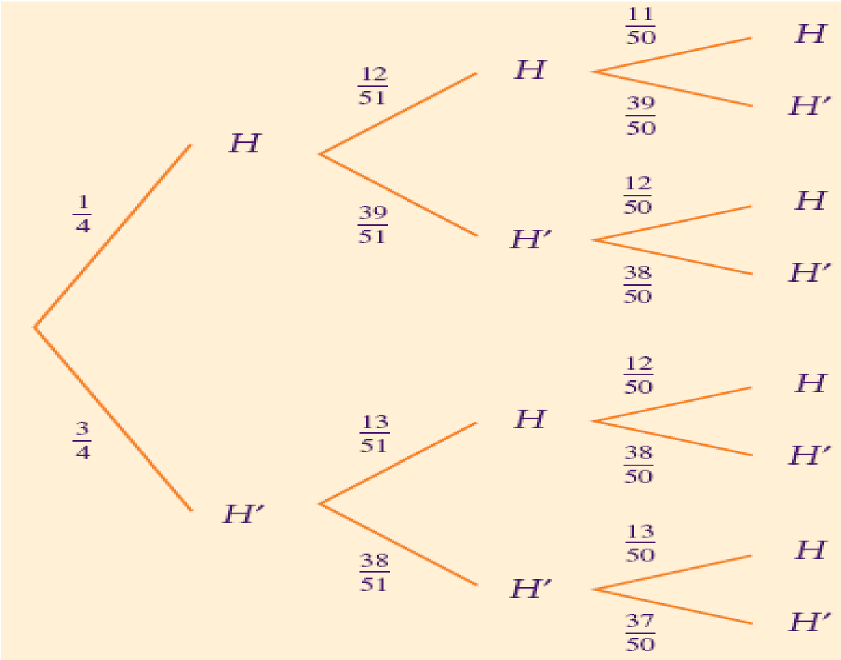
$\Pr(H') = {}^{39}/_{51}$

If a **non-head was drawn first** (13 of the 51) cards left are heads.

$\Pr(H) = {}^{13}/_{51}$

$\Pr(H') = {}^{38}/_{51}$

Similarly for the third draw, the probabilities will depend on what cards have been drawn before. The tree diagram is shown below –



From the diagram there is only one event that satisfies the requirements of three heads in a row (HHH)

$\Pr(HHH) = {}^{13}/_{52} \times {}^{12}/_{51} \times {}^{11}/_{50} = {}^{11}/_{850} \approx 0.013\ (1.3\%)$

Notes	Probability tables – can only be used with two events Tree Diagrams-very versatile but take up a lot of space

Conditional Probability

$\Pr(A/B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$	This means the probability of event A occurring **given that** event B has occurred. When you see that word or this type of situation you must proceed with caution. Having a certain condition means that the sample space is reduced to just events

	with that condition. When identifying a conditional probability problem we are looking for the words **IF** or **GIVEN THAT** $\Pr(A/B)$ can be read as find the probability of A given B i.e. probability of A occurs given or knowing that B has occurred.
Example 8	Let examine the previous problem of tossing a coin 3 times. Consider the following events. A = at least 2 heads H = first toss is a head Now let's list the event space or all the outcomes; HHH, HHT, HTH, HTT, THH, THT, TTH, TTT Pr (A) = Pr (2 heads) or Pr (3heads) = $\frac{3}{8}+\frac{1}{8}=\frac{4}{8}$ $\Pr(A/B)=\frac{\Pr(A\cap B)}{\Pr(B)}$ $\rightarrow \Pr(A/B)=\frac{\frac{3}{8}}{\frac{4}{8}}=\frac{3}{4}$ Notice the event space of A and H Pr (H) = $\frac{4}{8}$ and $\Pr(A\cap B)=\frac{3}{8}$ So conditional probability is concerned with a reduced sample space
Example 9	Final example with an explanation of conditional probability Ted tosses a die and asks Georgia the number. Georgia's chance of guessing the correct number is $\frac{1}{6}$ Now Ted gives Georgia a clue by telling her that an even number has turned up. Georgia concludes that a 2 or 4 or 6 has turned up, so her chance of guessing the

number now is $\frac{1}{3}$

Such a problem could be worded as follows:

What is the probability that **IF**, the throw of a die produces an even number, that number is divisible by three?

Notice that is different from the following question: What is the probability that the number is even and divisible by 3? (Answer; $\frac{1}{6}$, no condition attached to this question)

Example 10

Two dice are tossed. What is the probability that the sum of the numbers exceeds 10 given a 5 with the first die?

Solution

It is conditional, did you notice the word ‘given’

Let A be a 5 with the first die
Let B denote ‘the sum of the numbers exceed 10’

Let look at the sample space

1,1	2,1	3,1	4,1	**5,1**	6,1
1,2	2,2	3,2	4,2	**5,2**	6,2
1,3	2,3	3,3	4,3	**5,3**	6,3
1,4	2,4	3,4	4,4	**5,4**	6,4
1,5	2,5	3,5	4,5	**5,5**	6,5
1,6	2,6	3,6	4,6	**5,6**	6,6

Notice the bold numbers that represents the first die with a 5

Pr (B/A) = $\frac{1}{6}$ just reading off the sample space and using the reduced space

Or we could use the formula

$$\Pr(B/A) = \frac{\Pr(B \cap A)}{\Pr(A)}$$

$$\Pr(A/B) = \frac{\frac{1}{36}}{\frac{6}{36}}$$

$$\rightarrow \frac{1}{6}$$

Sometimes it is better to show the sample space and find the answer rather than use the formula!

INDEPENDENT EVENTS	2 events are independent if Pr (A/B) = Pr (A), so whether or not B has occurred it has no effect on the probability of B occurring $\Rightarrow \Pr(A \cap B) = \Pr(A) \bullet \Pr(B)$ (We say A and B are independent of each other)
Example 11	The probability of Ted remembering to do his homework is 0.7 while the probability of Zen remembering to do her homework is 0.4. If these two events are independent then what is the probability a) Both will do their homework? Solution Let T be Ted and he does his homework while let Z be Zen and she does her homework therefore $\Pr(T \cap Z) = \Pr(T) \times \Pr(Z)$ $= 0.7 \times 0.4$ $= 0.28$ b) Ted will do his homework while Zen will not do her homework $\Pr(T \cap Z') = \Pr(T) \times \Pr(Z')$ $\rightarrow = 0.7 \times 0.6$ $\rightarrow = 0.42$ Remember $\Pr(Z') = 1 - \Pr(Z)$ $\rightarrow 1 - .04$ $= 0.6$
Various Probability Questions solved	
Example-12	If Pr (A) =0.52, $\Pr(A \cap B)$ =0.25 and $\Pr(A \cup B)$ =0.7 Find Pr (B) Solution $\Pr(A \bigcup B) = \Pr(A) + \Pr(B) - \Pr(A \bigcap B)$ $0.7 = 0.52 + \Pr(B) - 0.25$ $\Pr(B) = 0.43$
Example-13	A coin is tossed twice. What is the total number of possible outcomes? Solution First toss $\rightarrow$ 2 possible outcomes, H or T Second toss $\rightarrow$ again each one of the above outcomes can also be associated with a H or T

	So 4 possible outcomes We could show this by drawing a tree diagram or just listing the different possibilities
Example 14	A die is tossed. For the 2 events A and B where A is getting an even number and B is getting a number less than 4, find the probability of A or B Solution Total possible numbers: 1, 2, 3,4,5,6 A = even numbers= 2, 4, 6 B = number less than 4 $\rightarrow$ 1, 2, 3 Pr (A) = $\frac{3}{6}$ and Pr (B) = $\frac{3}{6}$ and $\Pr(A\cap B)=\frac{1}{6}$ $\Pr(A\bigcup B)=\Pr(A)+\Pr(B)-\Pr(A\bigcap B)$ $\Pr(A\bigcup B)=\frac{3}{6}+\frac{3}{6}-\frac{1}{6}$ $\rightarrow \frac{5}{6}$
Example 15	A coin is tossed 3 times Let A be the event ‘at least 2 tails’ Let B be the event 3 heads or 3 tails Let C be the event ‘at least one tail’ Which of the following is independent a) A and B b) A and C a) A and B Draw a tree diagram (it looks exactly like the previous 3 toss example) a) A and B A = at least 2 tails (means 2 tails or more) $\rightarrow$ HTT, THT, TTH, TTT Pr (A) = $\frac{4}{8}$ Now B $\rightarrow$ 3 heads or 3 tails $\rightarrow$ HHH, TTT Pr (B) = $\frac{2}{8}$ $A\cap B=$ TTT

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$\Pr(A \cap B) = \frac{1}{8}$

Now if there were independent then $\Pr(A \cap B) = \Pr(A) \bullet \Pr(B)$

$\Pr(A \cap B) = \Pr(A) \bullet \Pr(B)$

$\Pr(A \cap B) = \frac{4}{8} \bullet \frac{2}{8}$

$\Pr(A \cap B) = \frac{1}{8}$

Notice that the two results agree therefore we can say that A and B are independent

b)

C $\rightarrow$ at least one tail $\rightarrow$ HHT, HTH, HTT, THH, THT, TTH, TTT

$\Pr(C) = \frac{7}{8}$

$A \cap C$ =HTT, THT, TTH, TTT

$\Pr(A \cap C) = \frac{4}{8}$

Now $\Pr(A \cap C) = \Pr(A) \bullet \Pr(C)$

$\Pr(A \cap C) = \Pr(A) \bullet \Pr(C)$

$\rightarrow = \frac{4}{8} \bullet \frac{7}{8}$

$\rightarrow = \frac{28}{64}$

Now these two results do not agree therefore we can say that A and C are not independent.

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<tr><td>Example 16</td><td>

Mike and Betty are a married couple aged 30. According to life tables, probabilities of their being alive in 40 years time are approximately 0.7 and 0.8. Calculate the probability that 40 years from now

a) Both will be alive
b) Neither of them will be alive
c) Only one will be alive
d) At least one will be alive

Solution

A tree diagram would be a good idea

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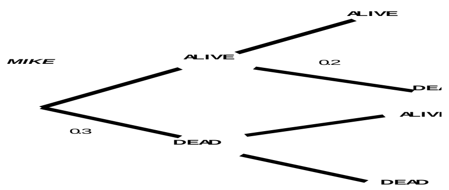


a)

Pr (Mike alive) = 0.7

Pr (Betty alive) =0.8

Events are independent therefore Pr (both alive) = 0.7*0.8=0.56

b)

Pr (Mike not alive) =0.3

Pr (Betty not alive) =0.2

Events are independent again therefore Pr (neither alive) = 0.2*0.3=0.06

c)

The statements ‘only one will be alive’ implies

that Mike is alive and not Betty or Betty alive and not Mike

Pr (Mike alive and not Betty) = 0.7*0.2=0.14

Pr (Betty alive and not Mike) =0.8*0.3=0.24

Mutually exclusive = 0.14+0.24=0.38

d)

Statement ‘at least one of them’ means 1 or 2 will be alive

Pr (at least one will be alive) = 1-Pr (neither will be alive)
=1-0.06
=0.94

Summary- You put down main ideas and expressions to be on the lookout for

Probability- Part-2

Discrete Probability and Binomial Probability

Discrete Probability Distributions

This type of probabilities does the following unique things

1. Gives the full list of possible values that *x* can take along with all the probabilities for each value
2. The sum of all probabilities must total 1

Consider the example below:

This is how a probability distribution table would look like if we tossed a die only once and obtain the numbers below:

x	1	2	3	4	5	6
Pr(X=*x*)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

If you add all the probabilities you would get 1

$$\rightarrow \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$$

$$\rightarrow \frac{6}{6}$$

$$\rightarrow 1$$

This function can then be called a discrete probability function

So then a discrete probability distribution is a description of a discrete random variable in terms of the individual's values that the variables can take and their associated probabilities.

REMEMBER

Discrete variables are variables which only take certain separate and well defined values, these are often number values

Also it is common to use upper letters such as X or Y for random variables. Lower case letters such as *x* or *y* are then used for a particular value of the random variable

Also a random variable can only assume countable number of distinct values it is called a discrete random variable

E.g. the outcomes of an experiment that measures the number of cars passing through a toll booth each hour

Or the outcomes of an experiment that measures the number of heads which occur when 3 coins are tossed

Example -1

At a large supermarket (Chook farmer's super chook market) during rush hour on Friday, the number of customers waiting in each of 10 checkout lines was counted.

Results

Number waiting	Frequency
2	1
3	2
4	3
5	3
6	1

Question: Construct a probability distribution for these results

Solution

Define random variable X, as the number of people waiting in a checkout line at rush hour on Friday

Thus X can take the values 2, 3,4,5,6 as can be seen from the table

Now we find the corresponding probabilities for each of these

X=2 $\to \frac{1}{10} \to 0.1$

Pr(X=2) means that there is 0.1 probability of the random variable X taking on the value of 2

X=3 $\to \frac{2}{10} \to 0.2$

X=4 $\to \frac{3}{10} \to 0.3$

x	2	3	4	5	6
Pr(X=x)	0.1	0.2	0.3	0.3	0.1

PRESENTATION OF PROBABILITY DISTRIBUTIONS

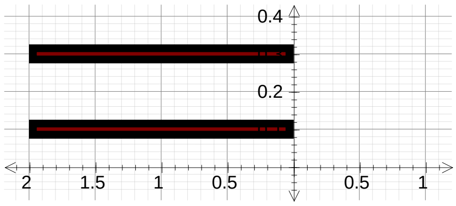
A probability distribution can be shown in a number of ways

A- The probability function can be stated i.e. $\Pr(x) = \frac{x}{10} \quad x \in \{1, 2, 3, 4\}$

B- It may be set out as in a table like the previous example

x	2	3	4	5	6
Pr(X=x)	0.1	0.2	0.3	0.3	0.1

C- Or it may be graphed as follows



Example -2

Use a table to set out the probability distribution as defined by the following function

$p(x) = \frac{x^2}{14}$ Where x $\in \{0, 1, 2, 3\}$

Graph the above distribution and verify that it is a probability distribution

Solution

$$p(0) = \frac{0^2}{14} = 0$$

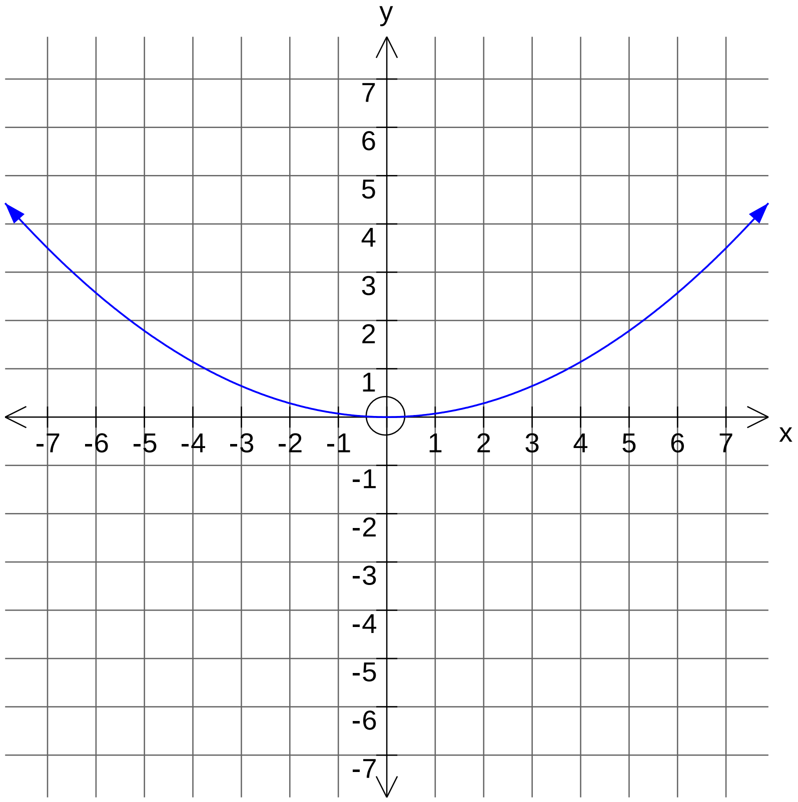
$$p(1) = \frac{1^2}{14} = \frac{1}{14}$$

$$p(2) = \frac{2^2}{14} = \frac{4}{14}$$

$$p(3) = \frac{3^2}{14} = \frac{9}{14}$$

Now creating the table

x	p(x)
0	0
1	$\frac{1}{14}$
2	$\frac{4}{14}$
3	$\frac{9}{14}$



To show that it is a probability distribution we add up all the probabilities and see if they add to 1

$$\sum p(x) = 0 + \frac{1}{14} + \frac{4}{14} + \frac{9}{14} = 1$$

And since the p(x) is ≥ 0 for all x, it is a probability function

EXPECTATION AND VARIANCE

Definition

Expectation (Mean Value)

The expected value of a discrete probability distribution can be likened to the long term average

Mean is showed by the symbol μ (mu pronounced) or $E(X)$

It is calculated as the sum of each possible value of x multiplied by its p(x) value for probability

$$\mu = E(X) = \sum x \Pr(X = x) = \sum xp(x)$$

The concept of expectation first came from gambling problems, where gamblers wished to know how much they could expect to win or lose in the long run.

Let's look again at a few examples

Example -3	A raffle offers a first prize of $1000, two second prizes of $300 each and twenty prizes of $15 each. If 10000 tickets at 50c each are sold, find the expected winnings of a person who buys one

	ticket? Solution Let x be the amount in dollars which can be won. Probability distribution table

x	P(x)
1000	$\frac{1}{10000}=0.0001$
300	$\frac{2}{10000}=0.0002$
15	$\frac{20}{10000}=0.002$
-.50 Notice –ve sign since losing money	$\frac{9977}{10000}=0.9977$

Remember the number of prizes is: 1+2+20=23
Only 10000 tickets sold take away 23 winning tickets $\rightarrow$ 9977 losing tickets

$E(X)=\sum x(p(x)$

$\rightarrow 1000\times 0.0001+300\times 0.0002+15\times 0.002+(-.50)\times 0.9977$

$\rightarrow -0.30885$

Therefore on average a person buying one ticket in this raffle will lose $0.31 or 31c.
Thus we could say that this is not a fair game.

A fair game would be one with an expected value of 0

For those going to casinos, they do not offer fair games…

Example -4

Consider the experiment of rolling a fair die. Let X= number of spots showing on uppermost face
Find E(X)
Solution
Let's draw table

x	1	2	3	4	5	6
Pr(X=x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Now $E(X)=\sum x(p(x)$

$(1\times\frac{1}{6})+(2\times\frac{1}{6})+(3\times\frac{1}{6})+(4\times\frac{1}{6})+(5\times\frac{1}{6})+(6\times\frac{1}{6})$

So $=\frac{21}{6}$

$=3\frac{1}{2}$

E(X) =3.5 This value is not possible to obtain for the variable x, however it is the number we expect on the long term average

Example -5

A roulette game, in which the wheel has 37 numbers on equally spaced slots. A player bets $1 on a number of his choice, that is he pays $1 to play the game. He receives $36 if he wins otherwise nothing. If X denotes the player's gain, what is his expected gain?

Solution

X	P(X=x)
-1	$\frac{36}{37}$
35 ($36-$1=35)	$\frac{1}{37}$

Now working out $E(X)=\sum x(p(x)$

$$E(X)=\sum xp(x)$$

$$E(X)=(-1)\times\frac{36}{37}+(35\times\frac{1}{37})=\frac{-1}{37}=-0.0270$$

Once again not a fair game since E(X) =0

If a player plays this game 1000 times he can expect to lose $1000\times-0.0270=\$27$

FURTHER PROPERTIES OF E(X)

To highlights some of these properties lets example this question

Example 6

Consider the following random variable of X, with the following distribution

x	1	2	3	4
Pr(X=x)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Is this a probability distribution function? Yes!

Why?

Because the probabilities add up to 1 $\rightarrow \frac{1}{5}+\frac{3}{10}+\frac{2}{5}+\frac{1}{10}$

Now let's find the following

a) E(X)

$$E(X)=(1\times\frac{1}{5})+(2\times\frac{3}{10})+(3\times\frac{2}{5})+(4\times\frac{1}{10})$$

$$E(X)=\frac{12}{5}$$

b) Find E (2X)

Solution

To find E (2X) we replace each value of x by 2x, thus we obtain the following

$$E(2X)=(2\times\frac{1}{5})+(4\times\frac{3}{10})+(6\times\frac{2}{5})+(8\times\frac{1}{10})$$

$$E(2X)=\frac{24}{5}$$

$$E(2X)=2\times\frac{12}{5}\rightarrow 2E(X)$$

c) Find E (2X+3)

Solution

Once again we do the same except this time we replace x by 2x+3

$$E(2X+3)=(5\times\frac{1}{5})+(7\times\frac{3}{10})+(9\times\frac{2}{5})+(11\times\frac{1}{10})$$

$$E(2X+3)=\frac{39}{5}$$

$$E(2X+3)=2\times\frac{12}{5}+3\rightarrow 2E(X)+3$$

The above example highlights the following properties

$$E(aX)=aE(X)$$

$$E(aX+b)=aE(X)+b$$

$$E(X+Y)=E(X)+E(Y)$$

VARIANCE AND STANDARD DEVIATION

On top of knowing the long run average (mean) it is also useful to have a measure of how close to the mean are the values.

VARIANCE

Variance is a measure of the spread of the probability distribution about its mean or expected value

$$Var(X) = E\left[(X-\mu)^2\right]$$

It is normal to show the variance with the following symbols: σ^2

STANDARE DEVIATION: $\sigma = \sqrt{\sigma^2}$

BOTH OF THE ABOVE MEASURE THE SPREAD

Variance as we mentioned is represented by:
$\sigma^2 = Var(X) = \sum (x-\mu)^2 \Pr(X=x)$
Standard deviation: $\sigma = \sqrt{Var(X)}$

$$Var(aX+b) = a^2 Var(X)$$

HINT: When doing questions asking for standard deviation, find the variance first and then take the square root to get σ. Also you might need to set up tables to find mean and variance.

SUMMING UP

$$\sigma^2 = Var(X) = E\left[(X-\mu)^2\right] = \sum (x-\mu)^2 \Pr(X=x) = E(X^2) - \mu^2$$

As always it is best to look at an example that will show how these formulas are used

Example -7

A random variable X has the probability distribution that follows

x	0	1	2	3
Pr(X=x)	0.3	0.1	0.25	0.35

Find the mean and the variance?

Let us set up the table so we might use the appropriate formula

x	Pr(X=x)	$E(X) = \sum x \Pr(X = x$	$Var(X) = (x-\mu)^2 \Pr(X=x)$	Another way of doing is by finding $x^2 \Pr(X=x)$
0	0.3	$0 \times 0.3 = 0$	$(0-1.65)^2 \times 0.3 = 0.81675$	0
1	0.1	$1 \times 0.1 = 0.1$	$(1-1.65)^2 \times 0.1 = 0.04225$	0.1
2	0.25	$2 \times 0.25 = 0.5$	$(2-1.65)^2 \times 0.25 = 0.030625$	1

3	0.35	$3 \times 0.35 = 1.05$	$(3-1.65)^2 \times 0.35 = 0.637875$	3.15
		Add all the above and we get $0+0.1+0.5+1.05=1.65$ Sum of this column is equal to the mean or E(X) or μ	Add all the above and we get σ^2 $\sigma^2 = 0.81675+0.04225+0.030625+0.637875$ $=1.5275$	Add the above and we get the following: 0+0.1+1+3.15=4.25 $\sigma^2 = 4.25 - \mu^2$ $\sigma^2 = 4.25 - 1.65^2$ $\sigma^2 = 1.5275$

FINAL NOTE ON MEANING OF STANDARD DEVIATION

It can be shown that most random variables, about 95% of the distribution lies within two standard deviations

$$\Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) \approx 0.95$$

What this means is the following: the probability that an observation lies within 2 standard deviations of the mean is approximately 0.95

Lets us consider one last example here to see this illustrated

Example 2

A bank has been engaged in a long term survey of the number of people who use its ATM machine during the lunch period of each day. The probability (relative frequency) distribution of X, the number of people who arrive at the machine per minute during lunch hour is as follows:

x	0	1	2	3	4
Pr(X=x)	0.36	0.38	0.18	0.06	0.02

Verify that 95% of the values of X lie within 2 standard deviations of the mean.

Solution

We must obviously find the mean and the standard deviation before we proceed to verify that the values lie within 95% of the mean

Step 1: Find mean

$$\mu = E(X) = (0 \times 0.36) + (1 \times 0.38) + (2 \times 0.18) + (4 \times 0.02)$$

$$\mu = E(X) = 1$$

Step 2: Find variance and then take the square root of it to obtain the standard deviation

$\sigma^2 = E(X^2) - [E(X)]^2 = E(X^2) - \mu^2$

From the above formula we need to find $E(X^2)$ and then find the variance

$E(X^2) = (0^2 \times 0.36) + (1^2 \times 0.38) + (2^2 \times 0.18) + (4^2 \times 0.02)$
$E(X^2) = 1.64$

Thus we can now use the formula $\sigma^2 = E(X^2) - \mu^2$

$\sigma^2 = E(X^2) - \mu^2$
$\sigma^2 = 1.64 - 1^2$
$\sigma^2 = 0.64$
$\rightarrow \sigma = \sqrt{0.64} = 0.8$

So we have obtained μ =1 and σ =0.8

Now finally we can start to use the formula $\Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) \approx 0.95$

Look at the formula and consider the left hand side of it namely: $\Pr(\mu - 2\sigma \le X \le \mu + 2\sigma)$

$\mu - 2\sigma = 1 - 2(0.8) = -0.6$ And $\mu + 2\sigma = 1 + 2(0.8) = 2.6$

$\Pr(-0.6 \le X \le 2.6) \rightarrow \Pr(0 \le X \le 2.6)$
$\rightarrow \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2)$
$\rightarrow 0.36 + 0.38 + 0.18$
$\rightarrow 0.92$

This is in fact close to 0.95 as required to show

Is there an easier way of doing this? Not really unless you use the graphics calculator

Note

$\Pr(\mu - \sigma \le X \le \mu + \sigma) \approx 0.68$

$\Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) \approx 0.95$

$\Pr(\mu - 3\sigma \le X \le \mu + 3\sigma) \approx 0.997$

BINOMIAL DISTRIBUTION

A binomial distribution experiment is characterized by the following features

- There are n identical trials
- Each trial is independent
- There are only 2 possible outcomes, which we call a success or a failure
- If the probability of success is p, then the probability of failure is 1-p
- It normally involves sampling with replacement

Formula

If x is number of successes in n trials with each trial having a probability of success of p then

$$\Pr(X = x) = {}^{n}C_{x} p^{x}(1-p)^{n-x}$$

Sometimes it can be written as $\Pr(X = x) = \binom{n}{x} p^{x}(1-p)^{n-x}$ where $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

Example -

Consider a set of 4 people whom we shall call A, B, C, D

How many combinations can we have taking those 2 at a time?

This is where we use the following: ${}^{n}C_{r}$ which basically means the number of combinations of *n* different objects taken *r* at a time.

${}^{4}C_{2} = \frac{4!}{2!(4-2)!} = \frac{4\times3\times2\times1}{2\times1(2\times1)} = \frac{24}{4} = 6$ So for the above example we can have 6 combinations

In how many ways can they be arranged taking them two at a time?

Solution

List possible arrangements:

AB	AC	AD	BC	BD	CD
BA	CA	DA	CB	DB	DC

So from the above we have 12 possible arrangements or otherwise called permutations. Notice that for example AB and BA are 2 different arrangements / permutations but only 1 combination!

So how many combinations or selections are their?

AB	AC	AD	BC	BD	CD

Only 6 selections or otherwise combinations

Important thing to remember is that:

Permutations $\rightarrow$ order is important $\rightarrow$ above example we have 12

Combinations $\rightarrow$ only concerned with selections $\rightarrow$ so we have 6 only selections or combinations

	Combinations $\times 2!$=Number of Permutations Remember $3! \to 3\times2\times1 \to 6$ $4! \to 4\times3\times2\times1 \to 24$ So general formula for combinations and permutations is given by the following $^nC_r = \frac{n!}{r!(n-r)!}$ Or $^nC_r = \frac{^nP_r}{r!}$ So finally with our example $^4C_2$ or it can be written as $\begin{pmatrix}4\\2\end{pmatrix}$ and it is read as 4 over 2

GENERAL FORMULA FOR BINOMIAL DISTRIBUTION

$$\Pr(X = x) = \begin{pmatrix}n\\x\end{pmatrix} p^x(1-p)^{n-x} \quad \text{Or} \quad \Pr(X = x) = {}^nC_x p^x(1-p)^{n-x}$$

Mean: $\mu = E(X) = np$

Variance: $np(1-p)$

Example	Dr Flick finds when experimenting with a particular drug, that it cures 7 patients out of 10. If 10 patients are given the drug find: a) Only 3 are cured It is binomial so lets us write out the formula $\Pr(X = x) = {}^nC_x p^x(1-p)^{n-x}$, where the probability of success is $\frac{7}{10} = 0.7$ n=10 and x= 3 cured $\Pr(X = 3) = {}^{10}C_3(0.7)^3(1-0.7)^{10-3}$ $\Pr(X = 3) = 0.009$ Remember: n= total number of trials x= number of success required p= probability of success per trial Continuing with the problem b) Find at least one of the patients is cured. Solution When you see that expression remember what it means: at least one could be 1 patient cured or 2 patients cured or 3 cured or 4 cured or 5 cured and so on. Obviously it would take us some time to figure out all those probabilities so we use a short

	cut $$\Pr(x \geq 1) = 1 - \Pr(x = 0)$$ $\Pr(x \geq 1) = 1 - \Pr(x = 0)$ $\Pr(x \geq 1) = 1 - {}^{10}C_0 (0.7)^0 (0.3)^{10}$ $\Pr(x \geq 1) = 0.999$ c) If Dr Flick sees 1000 patients, how many patients are expected to be cured? Solution Notice the expression: expected which should makes us remember the formula $\mu = E(X) = np$ $\mu = E(X) = np = 1000 \times 0.7 = 700$ Patients expected to be cured
Example	Ted kicks more than 4 goals in 60% of his football matches. In the next 5 matches, that he plays, what is the probability that he will kick more than 4 goals in a) Exactly 2 matches b) at least 3 matches Solution Probability questions can become extremely difficult because they are most worded questions and you need to be able to understand and extract as it were the information out of it. Notice firstly it is a binomial since they are only 2 outcomes; Kicks more than 4 goals (which we will call a success) or does not kick more than 4 goals Probability kicks more than 4 goals= $60\% = \frac{60}{100} \rightarrow 0.60$ Let n= 5 matches X= number of times kicks more than 4 goals in 5 matches a) $\Pr(X = 2) = {}^5C_2 (0.6)^2 (1-0.6)^{5-2}$ $\Pr(X = 2) = {}^5C_2 (0.6)^2 (1-0.6)^3 \rightarrow 0.2304$ We obviously can use our Ti83plus binompdf (5, 0.6, 2) =0.2304 (quick!) b) $\Pr(X \geq 3) = \Pr(X = 3) + \Pr(X = 4) + \Pr(X = 5)$ $\Pr(X \geq 3) = {}^5C_3 (0.6)^3 (1-0.6)^2 + {}^5C_4 (0.6)^4 (1-0.6)^1 + {}^5C_5 (0.6)^5 (1-0.6)^0$ $\Pr(X \geq 3) = 0.68256$ Or use the graphics calculator 1- binomcdf (5, 0.6, 2) =0.68256

Probability- Part 3

Continuous Probability and Normal Distribution Function

Previously we mentioned that any probability distribution the probability of an event happening must have a value between zero and 1.
We spoke about discrete probability functions that had very specific events such as tossing a coin and obtaining a head or tail, it is a specific event and it has a specific probability, but what about events that are continuous such as the height of people. This event does not have a specific height but includes a range of values.

So we can have a continuous variable whose values can include a range of real numbers. The probability that describes these types of events is called a probability density function. These probability density functions have certain characteristics as we shall see.

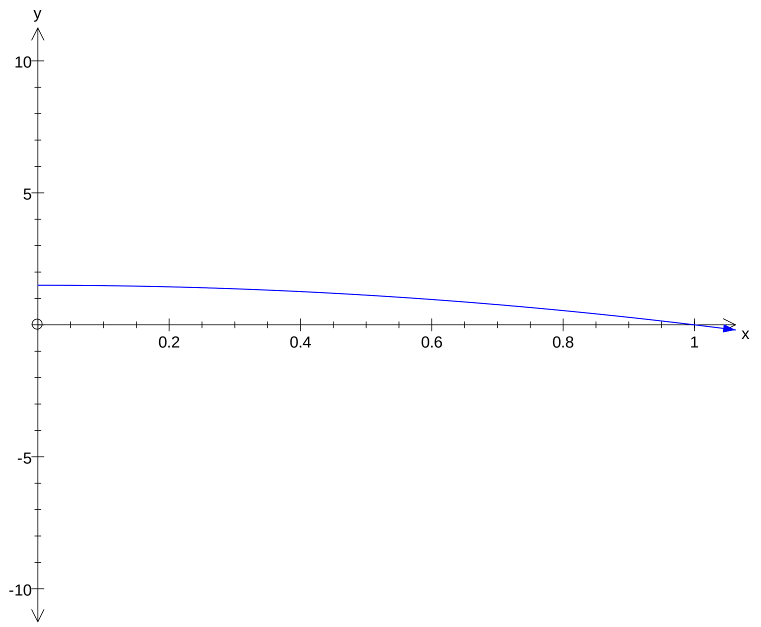
Probability Density Functions	Conditions for a probability density function $f(x) \geq 0$ for all real values of x The total area under the graph of $f(x)$ is 1 in other words $\int_{-\infty}^{\infty} f(x)dx = 1$

Example
Consider the function with the following characteristics

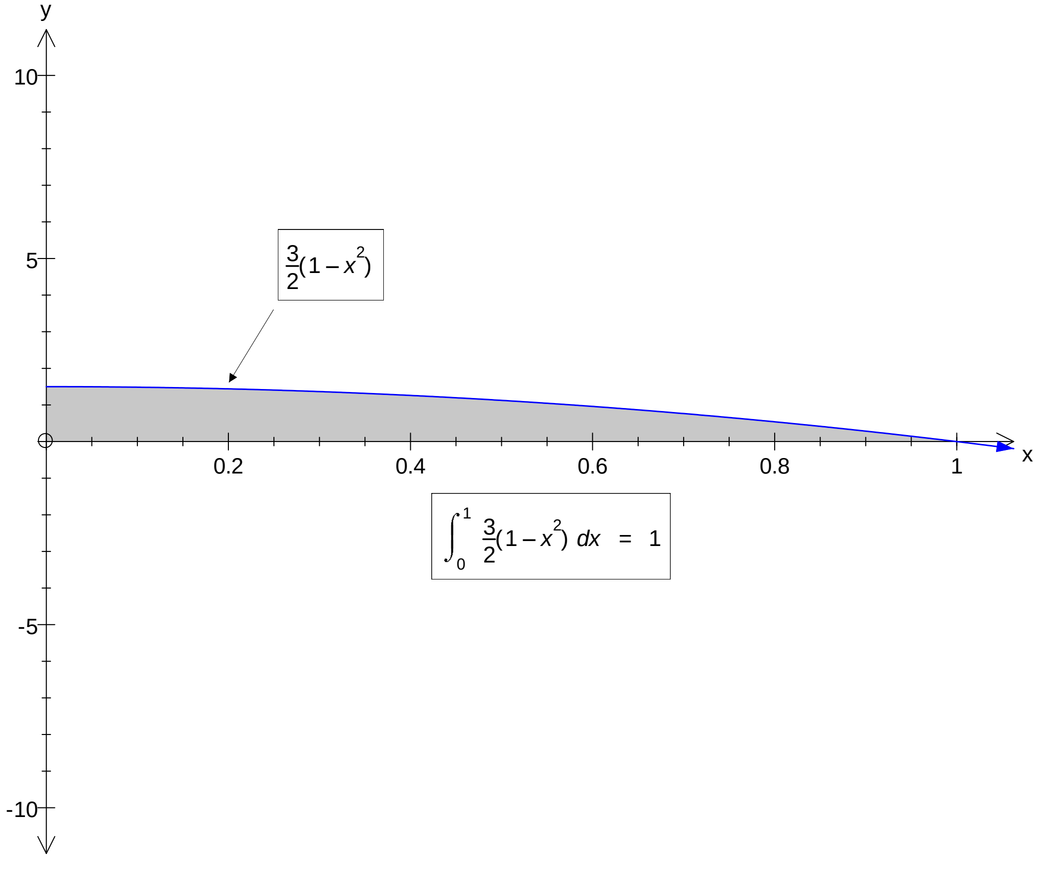
$$f(x) = \begin{cases} \frac{3}{2}(1-x^2) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that it is a probability density function
b) Find $\Pr(0.2 \leq x \leq 0.4)$
c) Find $\Pr(x < 0.5)$

Step-1- Sketch the graph to see how it looks



Now we need to find the area between 0 and 1 and see if the area comes to 1. If the area comes to 1 then this is a probability density function.



Now let us use calculus to work out the area

$$\int_0^1 \frac{3}{2}(1-x^2)dx = \frac{3}{2}\left(x - \frac{x^3}{3}\right)_0^1 = \frac{3}{2}\left(1-\frac{1}{3}\right) = \frac{3}{2}\left(\frac{2}{3}\right) = 1$$

So this function meets both conditions, namely

a) $f(x) \geq 0$ for all real values of x

b) $\int_{-\infty}^{\infty} f(x)dx = 1$

Now lets us work out the next part of the question, namely to find $\Pr(0.2 \leq x \leq 0.4)$

Solution

Just find the area under the graph between 0.2 and 0.4

$$\int_{0.2}^{0.4} \frac{3}{2}(1-x^2)dx = \frac{3}{2}\left(x - \frac{x^3}{3}\right)_{0.2}^{0.4} = \frac{3}{2}\left(\left(0.4 - \frac{0.4^3}{3}\right) - \left(0.2 - \frac{0.2^3}{3}\right)\right) = 0.272$$

a) Solution to part 3 , find $\Pr(x < 0.5)$

Once again find the area between 0 and 0.5

$$\int_0^{0.5} \frac{3}{2}(1-x^2)dx = \frac{3}{2}\left(x - \frac{x^3}{3}\right)_0^{0.5} = \frac{3}{2}\left(\left(0.5 - \frac{0.5^3}{3}\right) - (0)\right) = 0.6875$$

So that is how we work out probabilities involving continuous function!

Comparing Discrete versus continuous random variables

Discrete	Continuous
Probability function $p(x) = \Pr(X = x)$	Probability density function $f(x)$
$p(x) \geq 0$	$f(x) \geq 0$
$\sum p(x) = 1$	$\int f(x)dx = 1$
$\Pr(x_1 \leq X \leq x_2) = \sum p(x)$	$\Pr(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f(x)dx$

In the discrete case, we find probabilities by summing $\left(\sum\right)$ probabilities while in the continuous case we find probabilities by integration $\left(\int\right)$

Remember- It is always a good idea to sketch the graph of a probability density function!

Properties of continuous functions	
The Mean- $\mu = E(X)$	$\mu = E(X) = \int_{-\infty}^{+\infty} xf(x)dx$
The Median-	The median is the halfway mark of the population. For a continuous random variable, the median is the value of m such that $\Pr(X \leq m) = \frac{1}{2}$ $\int_a^m xf(x)dx = \frac{1}{2}$
The Mode	The mode is the most common value. For a continuous random variable, this will be the value M such that $f(M) \geq f(x)$ for all values of x There can be more than one mode. The mode occurs at the value or values of x for which the probability density function is a maximum. This can be found by using the derivative of the $f(x)$ or by sketching the graph and finding the maximum.
Variance	Variance $\sigma^2 = Var(X)$ is a measure of the deviation from the mean or average.

	$Var(X) = E(X^2) - \mu^2 = \int_{-\infty}^{+\infty} x^2 f(x)dx - \mu^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f($ Usually we use the graphics calculator and use the expression $Var(X) = E(X^2) - \mu^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx$
Standard deviation	$sd(X) = \sigma = \sqrt{Var(X)}$
Real application	

The temperature, X degrees Celsius, inside a refrigerator has been found to have a probability density function $f(x) = \frac{1}{k(\pi-2)} x\cos\left(\frac{x}{4}\right)$, $0 \le x \le 2\pi$, and zero elsewhere.

a) Find the value of k
b) Find the probability that the temperature inside the refrigerator is above $5^{\circ}C$
c) Calculate the average temperature inside the refrigerator correct to 2 decimal places.

Solution
a)
Since we are told that it is a probability density function then the area of this function must equal 1

$$\frac{1}{k(\pi-2)} \int_0^{2\pi} x\cos\left(\frac{x}{4}\right)dx = 1$$

We can use the power of the calculator to find that $k = 8$

b)

$$\Pr(X > 5) = \frac{1}{8(\pi-2)} \int_5^{2\pi} x\cos\left(\frac{x}{4}\right)dx = 0.1213$$

c)
We need to find the mean, so we proceed as follows

$$\mu = \frac{1}{8(\pi-2)} \int_0^{2\pi} x^2\cos\left(\frac{x}{4}\right)dx = 3.28$$

Normal Distribution

What is it?	It is a continuous distribution function that has very specific properties. It fulfills the probability density functions characteristics and its equation is below. $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

The equation of the normal distribution is very complex and it is based on the following formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

This graph has the following shape

To calculate the probability we find the area under this graph between a and b

$\Pr(a < x < b) = \int_a^b f(x)$ And as you can imagine this integral is just too difficult to use each time we are after the probabilities.

So what has happen is this, the normal curve is standardized so that μ =0 and σ =1

Now the formula becomes something that looks like this $g(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}z^2}$

Mathematicians have then gone and produced tables so that we can find the probabilities.
Of course it would be amiss of me to say that our graphics calculators can do the work of these tables faster as we say see soon.

But you must stay with me, since the explanation is important as some questions will require you to find the probabilities using the method that will be outlined below.

To standardize x values to Z we use the following formula, $Z = \frac{x-\mu}{\sigma}$, where x= data in the particular problem in question.

A positive Z value means the data is above the mean
A negative Z value indicates that the data is below the mean
Standard values of Z have the following characteristics μ =0 and σ =1

IMPORTANT

Calculations involving the normal distribution are either done using normal distribution tables or using the distribution function on Ti83 calculators

The tables are set for the standard normal and always give us the area below a positive Z value

On a Ti83 calculator, the probability that a value lies within two limits is given by entering the command

normalcdf (lower limit, upper limit, mean, standard deviation)
[If mean and standard deviation are left out calculator assumes μ =0 and σ =1]

Example 1

Find Pr (Z<1.32)

Solution

Using the graphics calculator (this is using an old texas instrument 83 calculator)

normalcdf (-1EE99, 1.32, 0, 1) =0.907

Or we could use the tables and look up the value 1.32 giving us 0.9066. Remember these tables are giving us the area of the shaded area of the diagram above

x	0	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9
	9																	
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	4	8	12	16	20	24	28	32	36
0.1	.5359									4	8	12	16	20	24	28	32	35
0.2	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	4	8	12	15	19	23	27	31	35
0.3	.5753									4	8	11	15	19	23	26	30	34
0.4	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	4	7	11	14	18	22	25	29	32
	.6141																	
0.5	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	3	7	10	14	17	21	24	27	31
0.6	.6517									3	6	10	13	16	19	23	26	29
0.7	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	3	6	9	12	15	18	21	24	27
0.8	.6879									3	6	8	11	14	17	19	22	25
0.9										3	5	8	10	13	15	18	20	23
	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190									
1.0	.7224									2	5	7	9	12	14	16	18	21
1.1	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	2	4	6	8	10	12	14	16	19
1.2	.7549									2	4	6	7	9	11	13	15	16
1.3	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	2	3	5	6	8	10	11	13	14
1.4	.7852									1	3	4	6	7	8	10	11	13
	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106									
1.5	.8133									1	2	4	5	6	7	8	10	11
1.6	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	1	2	3	4	5	6	7	8	9
1.7	.8389									1	2	3	3	4	5	6	7	8
1.8										1	1	2	3	4	4	5	6	6
1.9	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	1	1	2	2	3	4	4	5	5
	.8621																	
2.0	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	0	1	1	2	2	3	3	4	4
2.1	.8830									0	1	1	2	2	2	3	3	4
2.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	0	1	1	1	2	2	2	3	3
2.3	.9015									0	1	1	1	1	2	2	2	2
2.4	.9032	.9049	**.9066**	.9082	.9099	.9115	.9131	.9147	.9162	0	0	1	1	1	1	1	2	2
	.9177																	

2.5	.9192 .9207 .9222 .9236 .9251 .9265 .9279 .9292 .9306	0 0 0 1 1 1 1 1 1
2.6	.9319	0 0 0 0 1 1 1 1 1
2.7		0 0 0 0 0 1 1 1 1
2.8	.9332 .9345 .9357 .9370 .9382 .9394 .9406 .9418 .9429	0 0 0 0 0 0 0 1 1
2.9	.9441	0 0 0 0 0 0 0 0 0
	.9452 .9463 .9474 .9484 .9495 .9505 .9515 .9525 .9535	
3.0	.9545	0 0 0 0 0 0 0 0 0
3.1	.9554 .9564 .9573 .9582 .9591 .9599 .9608 .9616 .9625	0 0 0 0 0 0 0 0 0
3.2	.9633	0 0 0 0 0 0 0 0 0
3.3	.9641 .9649 .9656 .9664 .9671 .9678 .9686 .9693 .9699	0 0 0 0 0 0 0 0 0
3.4	.9706	0 0 0 0 0 0 0 0 0
	.9713 .9719 .9726 .9732 .9738 .9744 .9750 .9756 .9761	
3.5	.9767	0 0 0 0 0 0 0 0 0
3.6		0 0 0 0 0 0 0 0 0
3.7	.9772 .9778 .9783 .9788 .9793 .9798 .9803 .9808 .9812	0 0 0 0 0 0 0 0 0
3.8	.9817	0 0 0 0 0 0 0 0 0
3.9	.9821 .9826 .9830 .9834 .9838 .9842 .9846 .9850 .9854	0 0 0 0 0 0 0 0 0
	.9857	
	.9861 .9864 .9868 .9871 .9875 .9878 .9881 .9884 .9887	
	.9890	
	.9893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913	
	.9916	
	.9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934	
	.9936	
	.9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951	
	.9952	
	.9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963	
	.9964	
	.9965 .9966 .9967 .9968 .9969 .9970 .9971 .9972 .9973	
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	1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000	
	1.0000	

We can often use symmetrical properties in working out negatives values

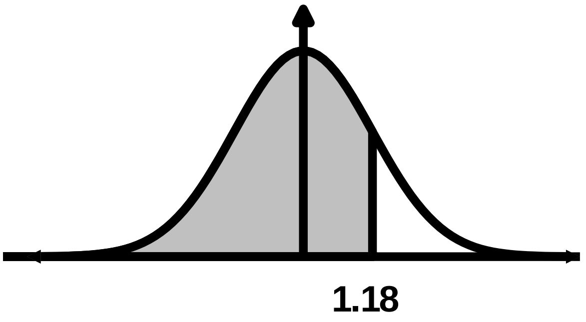
Look at this example below

Example 2

Pr (Z<-1.18)

Always draw a rough diagram of the normal distribution graph

Now we cannot use the table to find negative values such as -1.18 so what we do is use simple symmetry



1-Pr (Z<1.18) =1-0.8810=0.119 (this give us the non shaded area in above graph)

or using the calculator normalcdf (-1EE99, -1.18, 0, 1) =0.119

Example 3

If X is a normal distribution with mean 20, standard deviation 5, find Pr(X>27.5)

Solution

Using the normalcdf (27.5, 1EE99, 20, 5) =0.0668 this is the quickest way of working it out!

Now for the long way...

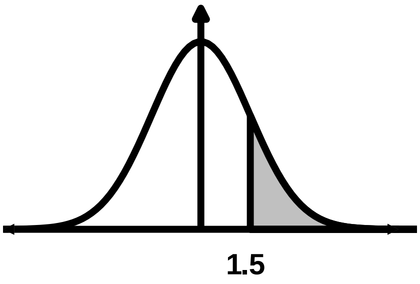
Notice it is not standardize so we must standardize it fist

$$Z = \frac{x-\mu}{\sigma} = \frac{27.5-20}{5} = 1.5$$

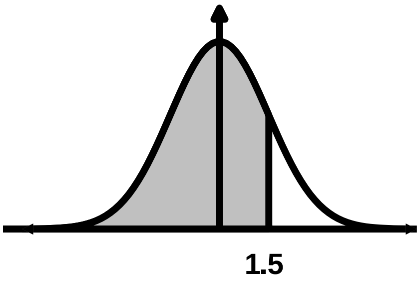
So Pr(X>27.5) =Pr (Z>1.5) now we can use the tables. But remember the tables only give us the probability of the area backwards i.e. that means < not >.

So we must use symmetry once again

Diagram



$\Pr(Z > 1.5) = 1 - \Pr(< 1.5)$ =1-.9332=0.0668



For the following see if you can draw the normal curves and work it out using the tables and your graphics calculator

Pr (Z<0) =0.5

Pr (Z<1) =0.8413

Pr (Z>1) =0.1587

Pr (Z<-2.3) =0.0107

One more example

<u>Example 4</u>

Pr (-1.246<Z<2.368)

Solution

Pr (Z<2.368)-Pr (Z<-1.246) $\rightarrow$

$0.9911 - [1 - \Pr(Z < 1.246)]$
$\rightarrow 0.9911 - [1 - .8936]$
$\rightarrow 0.8847$

As you can see from the above there is always two ways at arriving at the answer using the calculator or using the tables!

Example 5

If X is a normal distribution variable with μ =100 and σ =5, find Pr (X< 109)

Solution

Remember two ways at arriving at the answer

Quick way

Normalcdf (-1EE99, 109, 100, 5) =0.9641

Long way

First standardize

$$Z = \frac{x - \mu}{\sigma} = \frac{109 - 100}{5} = 1.8$$

$\therefore$ Pr (X< 109) = Pr (Z< 1.8) =0.9641 from the tables

INVERSE QUESTIONS

These types of questions have this format:

If you are given the probability and you want to find the number

$\Pr(Z < a) = 0.9$
Solution

Look up 0.9 using the inverse normal tables and work backwards

So lets say that μ =5and σ =3,

Then we have the following from the inverse tables Z=1.282

Now using the formula

$$Z = \frac{x-\mu}{\sigma}$$

$$1.282 = \frac{x-5}{3}$$

$$1.282 \times 3 = x - 5$$

$$x = 8.846$$

Finally we can use the normal distribution to approximate the binomial distribution if n is large (>30)

Example 1

A die is thrown 50 times. What is the probability that more than 15 sixes will be rolled?

Solution

Obviously it is a binomial since only 2 outcomes and probability of obtaining a six does not change

Pr(X>15) =Pr(X=16) + Pr(X=17) + Pr(X=18) + Pr(X=19) +……..+ Pr(X=50)

Pr(X>15) =1-Pr(X=15) =1 – binomcdf (50, $\frac{1}{6}$,15) =0.0057

Or using normal

$$\mu = np = 50 \times \frac{1}{6} = \frac{25}{3}$$

$$\sigma = \sqrt{50 \times \frac{1}{6}} \times \frac{5}{6}$$

$$\sigma = 2.6352$$

Pr(X>15) = normal cdf (15, 1EE99, $\frac{25}{3}$, 2.6352)=0.0057

So if n is large then the binomial can be approximated by the normal distribution

DETERMINING THE TYPE OF PROBABILITY DISTRIBUTION QUESTION

Easiest to pick the Normal distribution

1. Continuous distribution
2. You will always be told that the variable is normally distributed with a given mean and a standard deviation

A variable will have Binomial distribution if the outcome can be categorized as either a success or a failure with the same probability of success (p) on each of a series of (n) repeated trials

The binomial distribution will often come up in a normal distribution problem and the key to picking this is to look for the value of n

Let's consider a problem to highlight this

Example 2

Problem

The weight of lemons is normally distributed with a mean of 100grams and a standard deviation of 15 grams. Small lemons are found to have a weight of less than 70grams.

a) Find the proportion of lemons that are small

Solution

This is a normal distribution

Pr (X<70) =?

Standardize firstly $Z = \dfrac{x-\mu}{\sigma} = \dfrac{70-100}{15} = -2$

Pr (X<70) =Pr (Z<-2) =0.0228

b) If 9 lemons are packed in boxes of 10, find the probability that 4 are small

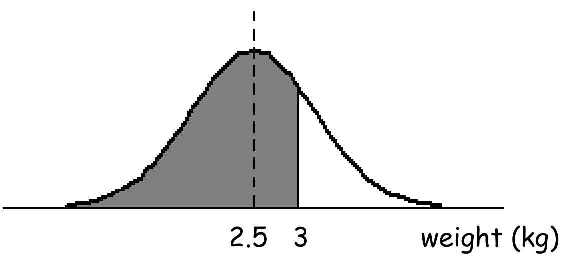
Solution

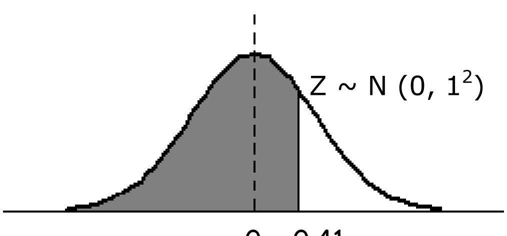
Notice how the question now becomes a binomial question!

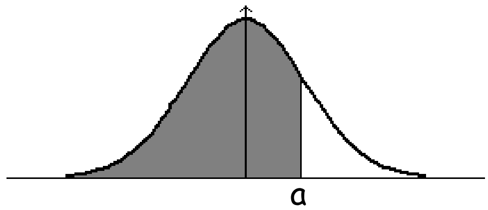
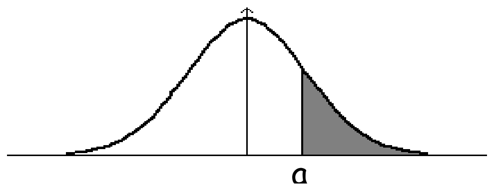
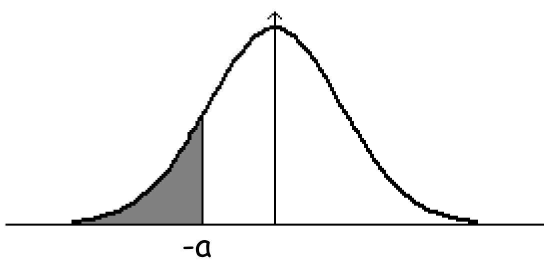
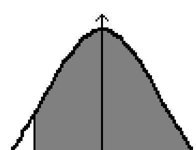
n=10 p (being small)=0.0228

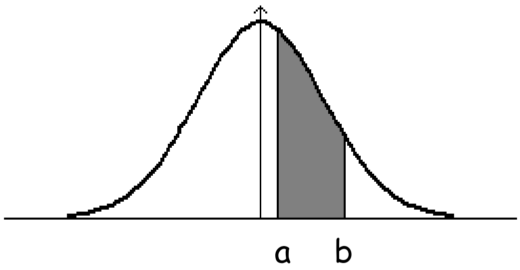
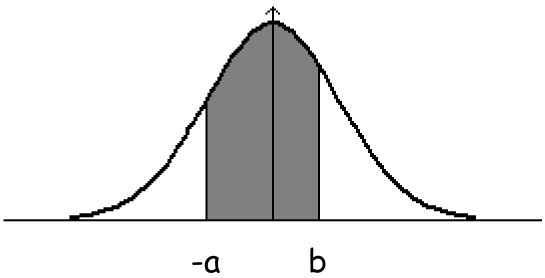
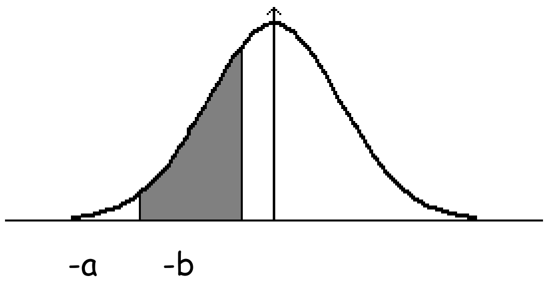
$$\Pr(X=4) = {}^{10}C_4(0.0228)^4(1-0.0228)^{10-4}$$
$$\Pr(X=4) = 0.000494$$

Using the Classpad calculator

More Examples on Normal Distribution	
Lets imagine that a cod fish has a mean of 2.5 kg and a standard deviation of 1.5, and it follows a normal distribution. Let us answer a few questions If we wanted to find the probability that the weight of a cod was less that 3 kg, i.e. P(X < 3), we would be interested in finding the area under the curve below 3 kg (the shaded area in the diagram).	Now always draw the normal distribution graph to see what we are after 
Step 1-Standardise- so we can use the tables or calculator Since this is very very difficult to calculate (the formula is very complicated to use), we transform the data to the Z variable because somebody has already worked out the	



probabilities for us (the table in the back of your text books. $P(X<3)=P\left(Z<\frac{X-\mu}{\sigma}\right)=P\left(Z<\frac{3-2.5}{\sqrt{1.5}}\right)$ $=P(Z<0.41)$ So we need to find $P(Z < 0.41)$. So the answer is 0.6591	
Make sure you understand the following rules. Make sure you are okay with (and understand) the following rules that you used to help you with the above questions	**Always** draw a diagram to help you find your probabilities
Rule 1- Use the calculator to work this out straightforwardly.	$P(Z < a)$ 
Rule 2	$P(Z > a) = 1 - Pr(Z<a)$ 
Rule 3	$P(Z < -a) = 1 - Pr(Z<a)$ 
Rule 4	$P(Z > -a) = Pr(Z<a)$ 

Rule 5	$P (a < Z < b) = Pr(Z<b) - Pr(Z<a)$ 
Rule 6	$P (-a < Z < b) = Pr(Z<b) - [1 - Pr(Z<a)]$ 
Rule 7	$P (-a < Z < -b) = [1 - Pr(Z<b)] - [1 - PZ<(a)]$ 

Probability- Part 4

Transition Matrices	
	Transition matrices are useful when we want to model situations for which the following conditions exist: -conditions or states are clearly defined -there is a transition from one state to the next state -the next state depends on the previous state -the conditional probabilities for each possible outcomes are the same on each occasion Transition matrices that follow the above conditions are known as Markov chains. The elements in transition matrices represent conditional probabilities, where each tells us the probability of an event occurring given the other event has already occurred. Here is a typical transition matrix

	Current States	
Next states	Event A	Event A'
Event B	$\Pr(B/A)$	$\Pr(B/A')$
Event B'	$\Pr(B'/A)$	$\Pr(B'/A')$

The associated transition matrix will look like this below:

$$T = \begin{bmatrix} \Pr(B/A) & \Pr(B/A') \\ \Pr(B'/A) & \Pr(B'/A') \end{bmatrix}$$

Notice that in the first column the elements

Event A
$\Pr(B/A)$
$\Pr(B'/A)$
The sum of the two above probabilities

	is 1

This makes sense because if A has occurred then B or the compliment of B has occurred, so that the sum of these elements is 1

Transition Matrix

The transition matrix multiplies the state matrix to form a new state matrix. A state matrix will look like this $s = \begin{bmatrix} \text{number in A} \\ \text{number in B} \end{bmatrix}$

Now the initial matrix we call s_0, which tells us how many objects were initially in each set.

$$s_0 = \begin{bmatrix} \text{number in A} \\ \text{number in B} \end{bmatrix}$$

How to calculate the state matrix

In general the state matrix after n transitions can be calculated using:

$$s_n = T^n s_0 \text{ or } s_n = T \times s_{n-1}$$

Be careful: when we multiply the transition matrix and the state matrix, the transition matrix must be placed before the state matrix.

Example-1

Imagine that the transition matrix is given by the following table below:

	Current States	
Next states	Event A	Event A'
Event B	0.6	0.3
Event B'	0.4	0.7

And lets us imagine that the initial state matrix , s_0 is given by the following:

$$s_0 = \begin{bmatrix} 300 \\ 500 \end{bmatrix}$$

Find s_1 and s_2 by hand and s_3 using the formula

Solution

To find s_1 we need to find $s_1 = T^1 s_0$

$$s_1 = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} \begin{bmatrix} 300 \\ 470 \end{bmatrix}$$

We multiply through now

<table>
<tr><td></td><td>

$s_1 = \begin{bmatrix} 0.6\times300+0.3\times500 \\ 0.4\times300+0.7\times500 \end{bmatrix}$

$s_1 = \begin{bmatrix} 339 \\ 461 \end{bmatrix}$

Please note that the total number of objects being considered is always equal to the total number of objects in the initial state matrix , which in this case is - $300+500=800$

Now let us work out $s_2 = T^2 s_0$

$s_2 = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}^2 \begin{bmatrix} 300 \\ 470 \end{bmatrix}$

Many ways of working this out. We could

$s_2 = T^1 s_1$ so we would get the following:

$s_2 = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} \begin{bmatrix} 330 \\ 500 \end{bmatrix}$

$s_2 = \begin{bmatrix} 0.6\times330+0.3\times500 \\ 0.4\times330+0.7\times500 \end{bmatrix}$

$s_2 = \begin{bmatrix} 339 \\ 461 \end{bmatrix}$

To find s_3 we use the formula $s_3 = T^3 s_0$

Therefore we have $s_3 = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}^3 \begin{bmatrix} 300 \\ 470 \end{bmatrix} = \begin{bmatrix} 341.7 \\ 458.3 \end{bmatrix}$

</td></tr>
<tr><td>**Steady state**</td><td>

Using the example from above we can conclude that the values in the state matrix may be approaching a limiting value as n continues to increase

$s_1 = \begin{bmatrix} 330 \\ 470 \end{bmatrix}$

$s_2 = \begin{bmatrix} 339 \\ 461 \end{bmatrix}$

$s_3 = \begin{bmatrix} 341.7 \\ 458.3 \end{bmatrix}$

$s_4 = \begin{bmatrix} 342.51 \\ 457.49 \end{bmatrix}$

$s_9 = \begin{bmatrix} 342.8563 \\ 457.1437 \end{bmatrix}$

</td></tr>
</table>

	Now if the state matrix s_n approaches a limit as $n \to \infty$ then the limit is referred to as the steady state. How do we find the steady state ? Let us imagine that the steady state is given by $\begin{bmatrix} A \\ B \end{bmatrix}$ Then multiplying the steady state matrix by T will not change its value since it has already reached a steady state $\begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$ Simplifying this matrix gives us the following: $0.3B = 0.4A$ $0.4A = 0.3B$ And since A + B = 800 we can then obtain a solution for B, namely being $B = 457.14$ and $A = 342.86$ Sometimes we can just put the equation into our graphics calculator and let the calculator do the work If a two state markov chain is described by a transition matrix $T = \begin{bmatrix} 1-a & b \\ a & 1-b \end{bmatrix}$ Then the steady state probabilities are $\Pr(X_n = 0) = \dfrac{b}{a+b}$ and $\Pr(X_n = 1) = \dfrac{a}{a+b}$
Example-2	A transition matrix is given by $T = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}$ and the initial state matrix is $s_0 = \begin{bmatrix} 4/15 \\ 6/15 \\ 5/15 \end{bmatrix}$ Find s_1, s_2 s_{12} and the steady state probabilities Solution Find s_1 $s_1 = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}\begin{bmatrix} 4/15 \\ 6/15 \\ 5/15 \end{bmatrix} = \begin{bmatrix} 0.3267 \\ 0.4133 \\ 0.2600 \end{bmatrix}$

	Find s_2 $$s_2 = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}^2 \begin{bmatrix} 4/15 \\ 6/15 \\ 5/15 \end{bmatrix} = \begin{bmatrix} 0.3547 \\ 0.3867 \\ 0.2587 \end{bmatrix}$$ Find s_{12} $$s_{12} = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}^{12} \begin{bmatrix} 4/15 \\ 6/15 \\ 5/15 \end{bmatrix} = \begin{bmatrix} 0.3571 \\ 0.3810 \\ 0.2619 \end{bmatrix}$$ To find the steady state matrix we need to solve the equation that results from the following matrix $$\begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.3 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$ Which gives us the following three equations to solve $$0.5p + 0.4q + 0.1r = p$$ $$0.2p + 0.4q + 0.6r = q$$ $$0.3p + 0.2q + 0.3r = r$$ Use the graphics calculator to solve $$p = \frac{5}{14}$$ $$q = \frac{8}{21}$$ $$r = \frac{11}{42}$$
Example-3	In Victoria there are three major newspapers, The Age, The Herald-Sun and the Australian. Readers tend to be fairly loyal to the newspapers they buy. Records in a particular country town have shown that of people who purchase a newspaper every day, 85% of people who buy The Age on one day will buy it the next day, 84% of people who buy The Herald-Sun on one day will buy it the next day and 79% of people who buy The Australian on one day will buy it the next day. For those who change which people they buy from one day to the next, purchases are summarized in the table below. On a particular day 800 copies of The Age, 1200 copies of The Herald-Sun and 500 copies of The Australian were sold. Assuming that the total number of all three newspapers sold each day remains constant, predict the number of copies of each paper which will be sold a) On the next day b) Two days later c) Per day in the long run. Table

	Current states		
Next states	Buying The Age on the first day	Buying The Herald –Sun on the first day	Buying The Australian on the first day
Buying The Age on the	0.85	0.09	0.12

next day			
Buying The Herald-Sun on the next day	0.06	0.84	0.09
Buying The Australian on the next day	0.09	0.07	0.79

Solution

$T = \begin{bmatrix} 0.85 & 0.09 & 0.12 \\ 0.06 & 0.84 & 0.09 \\ 0.09 & 0.07 & 0.79 \end{bmatrix}$ and the initial state matrix is $s_0 = \begin{bmatrix} 800 \\ 1200 \\ 500 \end{bmatrix}$

a) On the next day

$s_1 = Ts_0$

$$s_0 = \begin{bmatrix} 0.85 & 0.09 & 0.12 \\ 0.06 & 0.84 & 0.09 \\ 0.09 & 0.07 & 0.79 \end{bmatrix} \begin{bmatrix} 800 \\ 1200 \\ 500 \end{bmatrix} = \begin{bmatrix} 848 \\ 1101 \\ 551 \end{bmatrix}$$

So on the next day, the number of copies sold is predicted to be 848 copies of The Age, 1101 copies of The Herald-Sun and 551 copies of The Australian

b) Two days later

$s_2 = Ts_1$

$$s_2 = \begin{bmatrix} 0.85 & 0.09 & 0.12 \\ 0.06 & 0.84 & 0.09 \\ 0.09 & 0.07 & 0.79 \end{bmatrix} \begin{bmatrix} 848 \\ 1101 \\ 551 \end{bmatrix} = \begin{bmatrix} 886.01 \\ 1025.1 \\ 588.68 \end{bmatrix}$$

So two days, the number of copies sold is predicted to be 886 copies of The Age, 1025 copies of The Herald-Sun and 589 copies of The Australian

c) Per day in the long run.

Let the steady state matrix by $\begin{bmatrix} A \\ B \\ C \end{bmatrix}$

Then we need to solve

$$\begin{bmatrix} 0.85 & 0.09 & 0.12 \\ 0.06 & 0.84 & 0.09 \\ 0.09 & 0.07 & 0.79 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

Solution works out to be as follows
The Age- 1025
The Herald-Sun-777
The Australian-698

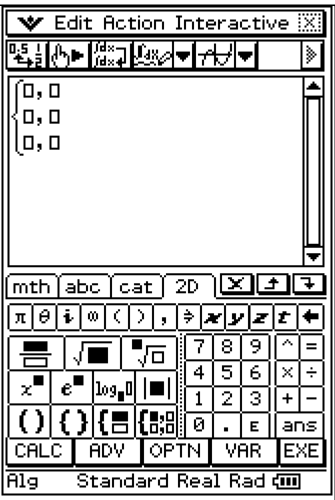
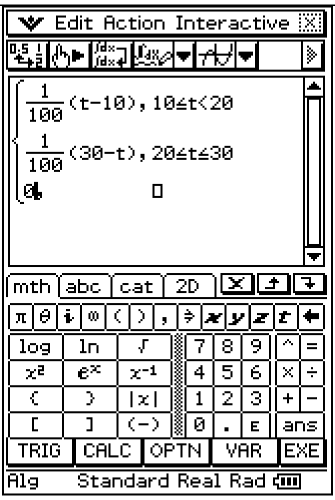
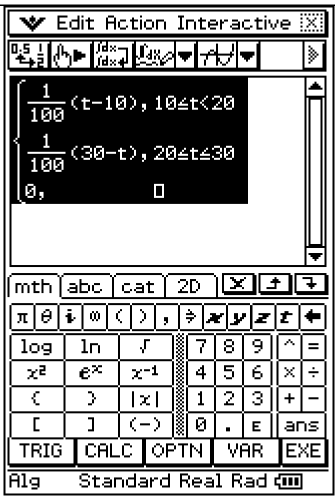
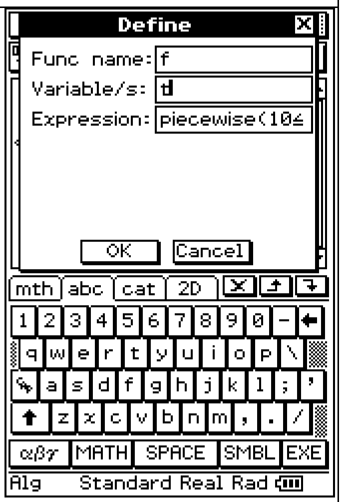
Probability-Classpad-examples

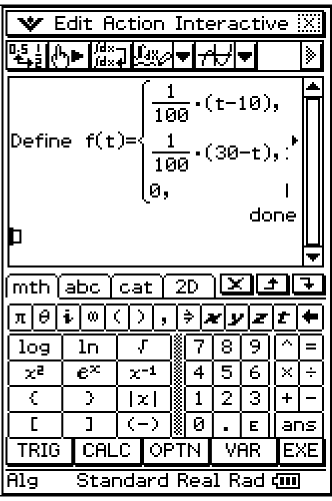
Probability Density Function

How do we find the probability given a piecewise probability density function?
Example:
The continuous random variable T, time in minutes has a probability density function, f, with the rule has follows:

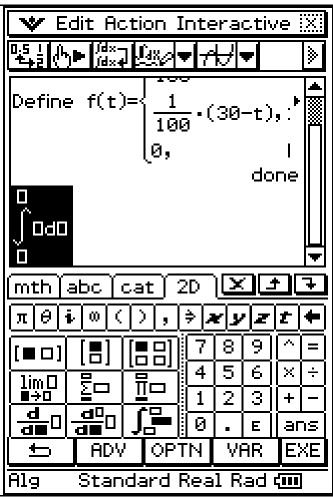
$$f(t)=\begin{cases}\frac{1}{100}(t-10) & \text{if } 10\le t<20\\ \frac{1}{100}(30-t) & \text{if } 20\le t<30\\ 0 & otherwise\end{cases}$$

Find the probability that $T<25$

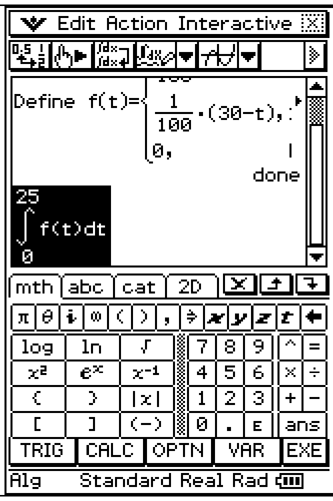
			
Go to Main screen Use the 2D and get the piecewise template Now tap twice will create a template with 3 elements	Enter the probability function Leaving a domain field empty means otherwise	Select it all and then go to Interactive Define Now define the equation calling it f	Make sure you change the variable to t Press OK



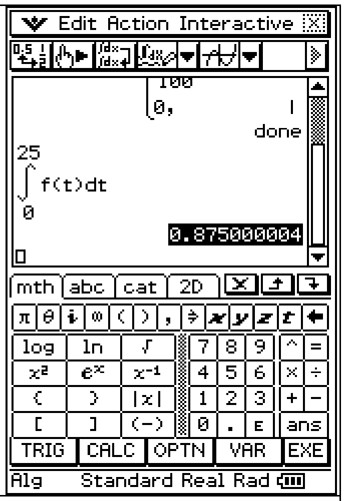
This is what you will see



Now go to 2D and go to the integration symbol



Put the variable in carefully
Remember variable t

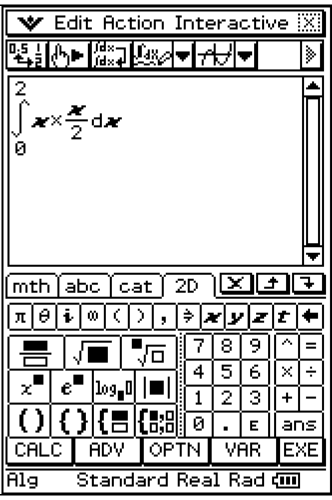


Press EXE gives the answer

Expected value and Median

How do we calculate the expected value and the median of a probability density function?
Example

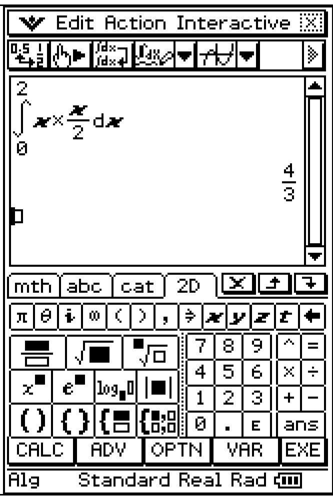
Find the median and the expected value of a random variable X that has a probability density function $f(x)=\frac{x}{2}$ for $0 \le x \le 2$



To find the expected value we need to use the following formula

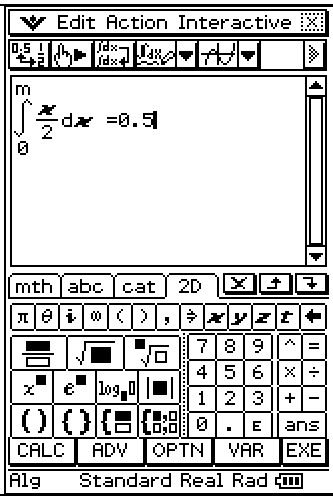
$$E(X)=\int_a^b xf(x)dx$$

So we go to 2D view and find the template for integration



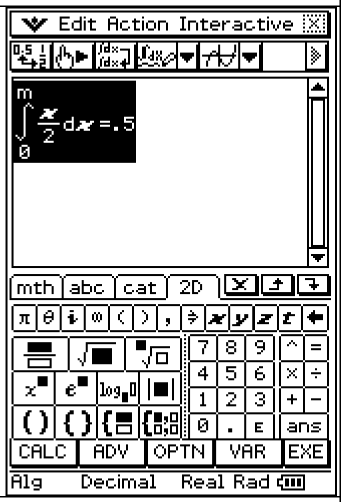
Press EXE and we will get the answer

So the answer is $\frac{4}{3}$

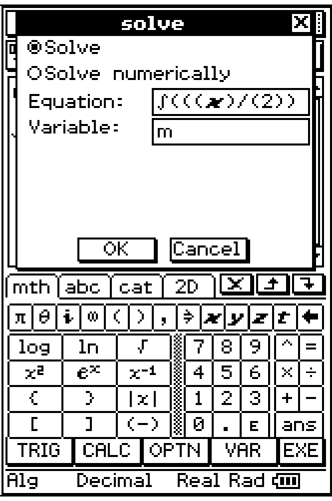
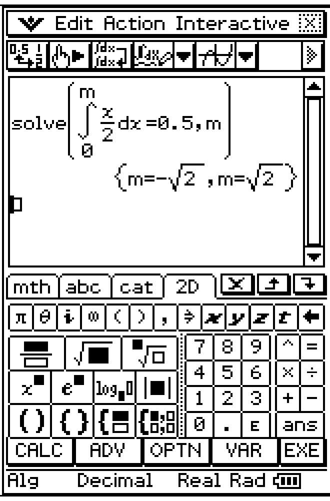
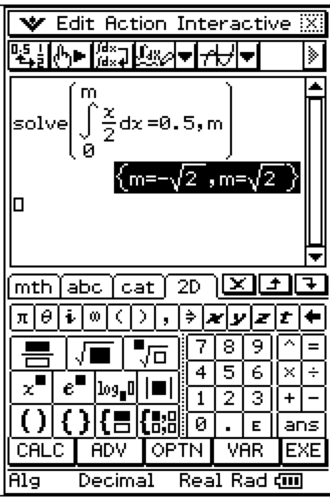
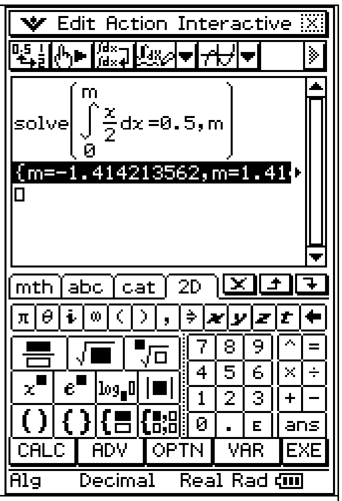


To find the median we need to find the value , m such as this equation is satisfied

BE VERY CAREFUL AFTER ENTERING x in dx we need to making sure the cursor is flashing all of the expression

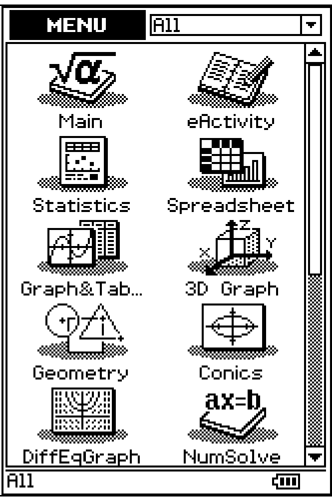
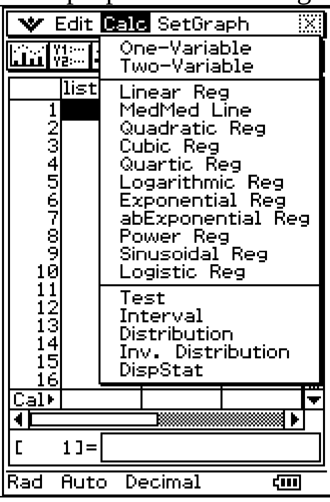
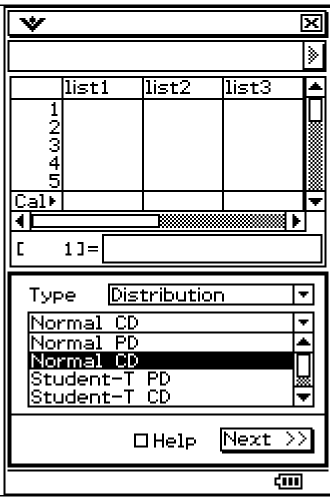
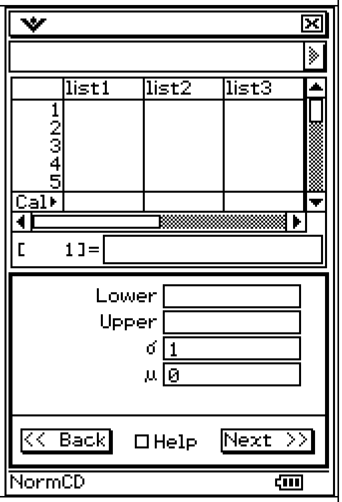


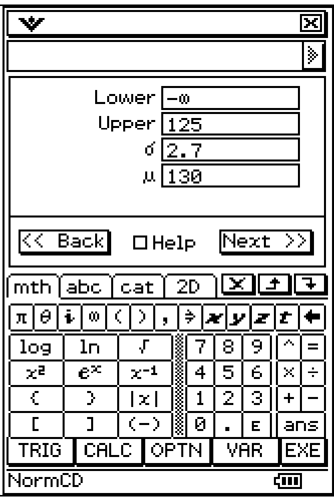
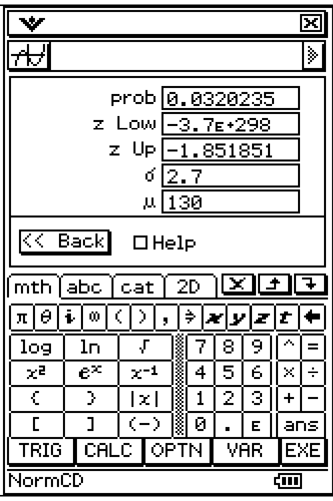
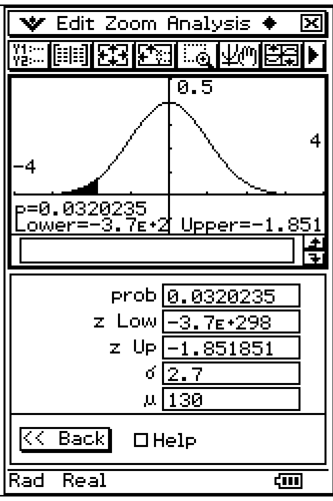
Press interactive
Equation
solve

			
Make sure you set the variable to m	And you can get the exact value or you can highlight it to get the decimal values	Highlighting the answer and pressing top fraction to decimal button give	Answer in decimal points.

Normal distribution Probability

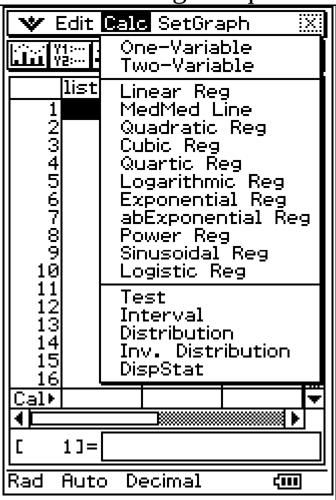
How do we calculate the probability with normal distribution?
Example: The height of girls under 14 years old in a survey are normally distributed with a mean of 130cm and standard deviation of 2.7 cm. What proportion of these girls are shorter than 125cm?

			
Launch the statistics application	Go to Calc Then go to Distribution G	Then go to the drop down selection and pick Normal CD Press Next	This is what you will see Put the information

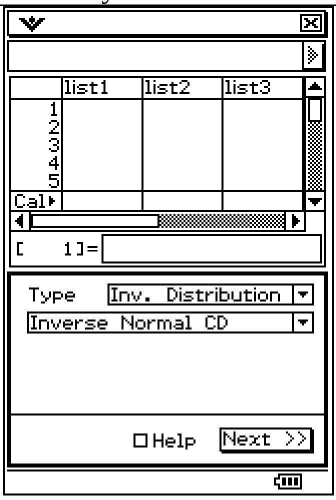
			
Lower - $-\infty$ Upper-125 $\sigma = 2.7$ $\mu = 130$ You might have to adjust your screen to Maths numbers appearing Press Next	The answer is there 0.0320235 We can see the result graphically by pressing the graph on the top left corner	This is the result which shows it graphically	

Inverse Normal Distribution

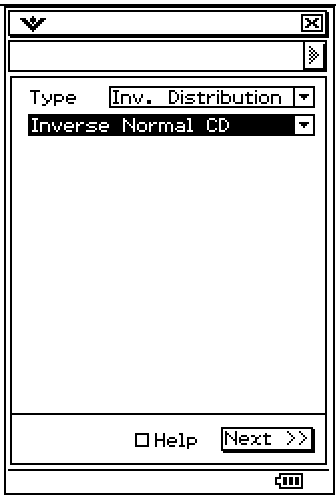
How do we calculate a boundary given a probability with a normal distribution.
For example using the previous question, say we are asked to find the minimum acceptable height if 35% of girls fail to meet the height requirement for entry into a amusement ride.



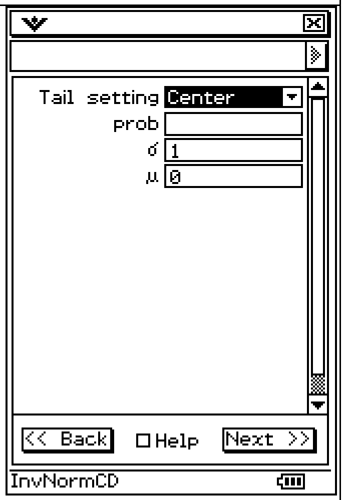
Go to statistics again
Go to Cal
Now click on Inv Distribution



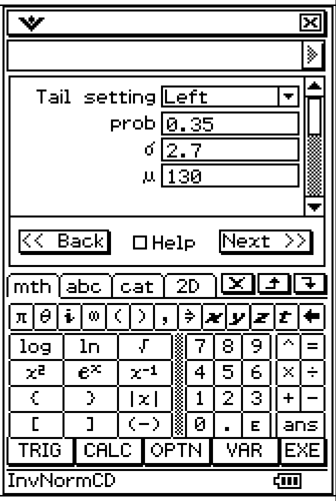
This is what you will see
Press bottom window and resize



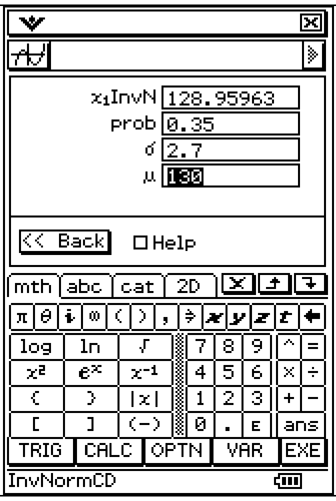
So we have resized
Now select inverse normal CD and press next



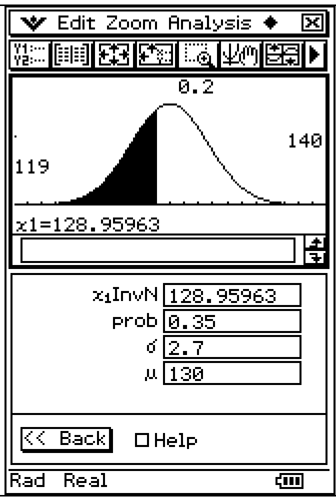
Notice we need to adjust the tail setting to the left
And enter the information we have



Tail setting-left
Prob-0.35
$\sigma = 2.7$
$\mu = 130$



And the answer is given for us
Cut off boundary 128.95
We can use graph to see it



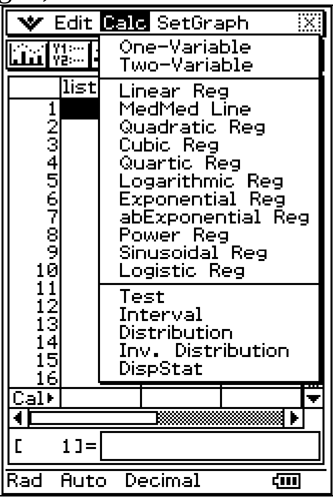
Which shows the answer clearly for us to see

Normal distribution-Mean and Standard Deviation

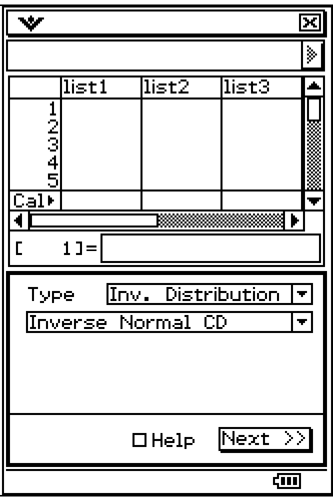
How do we calculate the unknown parameters for a normal distribution

Example

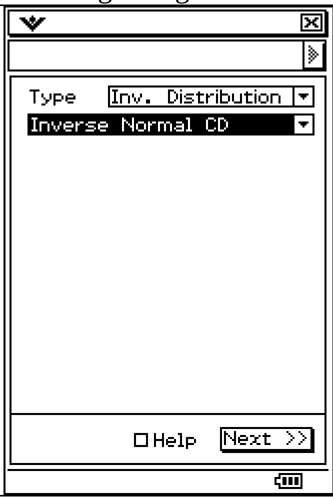
The weights of a certain orange are normally distributed. If 5% weigh more than 30g and 10% weigh less than 15 gra., find the mean and standard deviation of the distribution of orange weights.



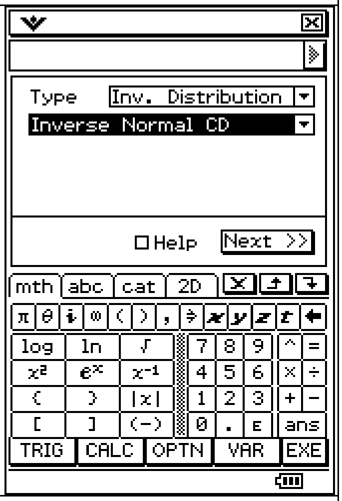
Go to statistics in menu
Then calc
Then Inv Distribution



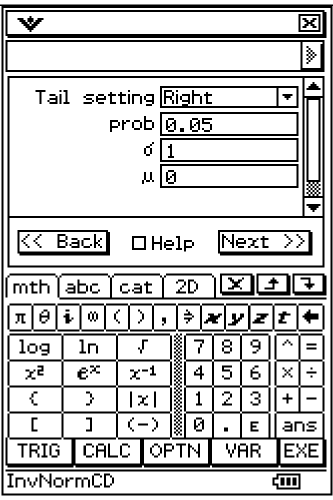
Resize screen by pressing the bottom part and then resize option



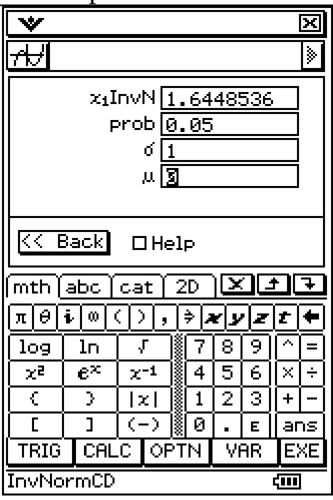
Put the math menu on the botton



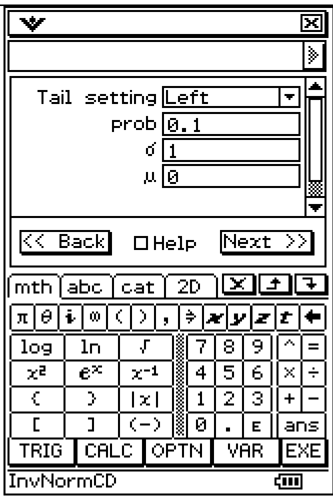
Now we press next



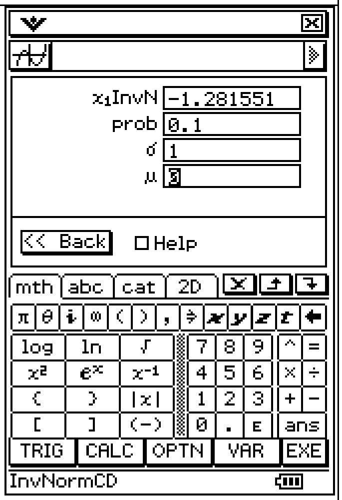
Set tail to right
Prob- 0.05
$\sigma = 1$
$\mu = 0$
Press Next



We get the first Z-score which is 1.64485



Now repeat but this time put the other bit of information
Set tail to left
Prob- 0.1
$\sigma = 1$
$\mu = 0$
Press Next

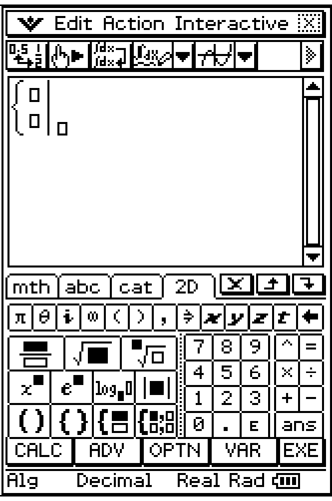
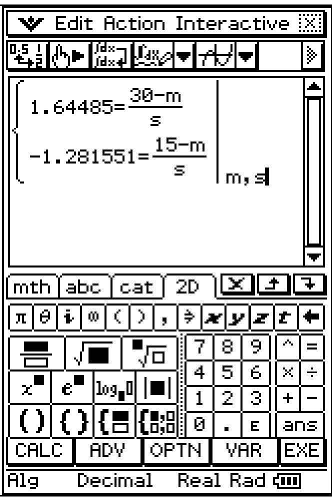
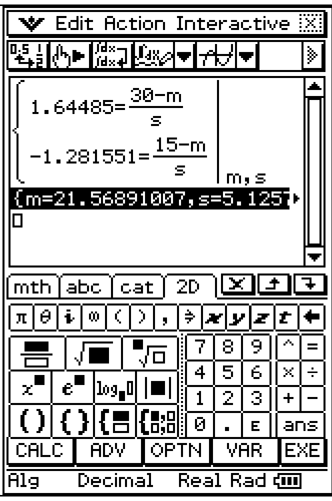


So we get the Z-score with this being -1.2815

Put these two values into equation form

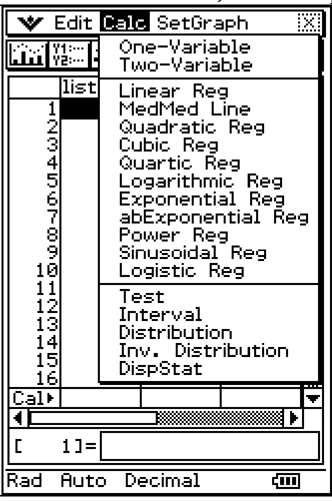
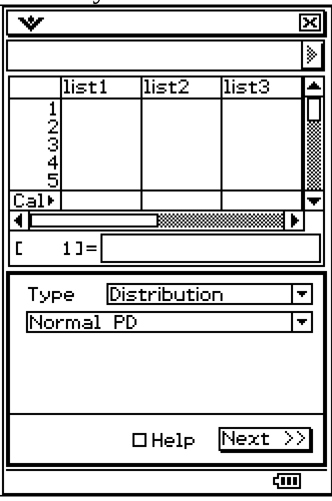
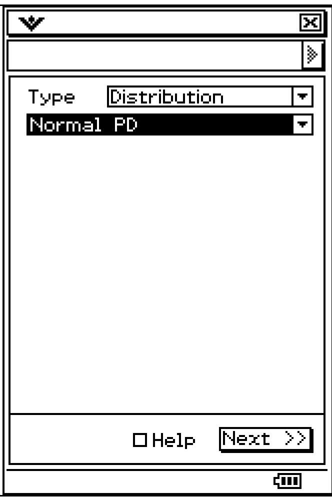
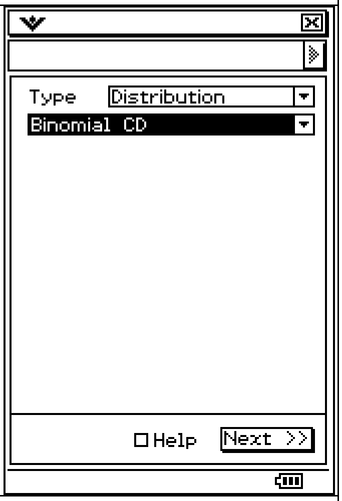
$$1.64485 = \frac{30-\mu}{\sigma}$$

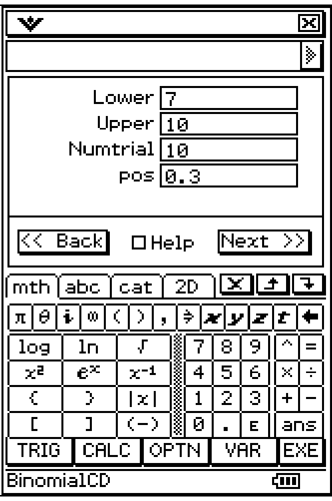
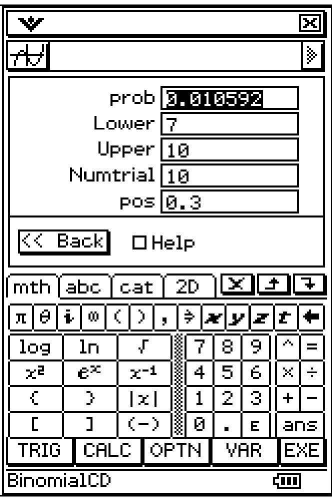
$$-1.281551 = \frac{15-\mu}{\sigma}$$

			
Go back to Main section and we need to put these two equation using 2D template	Put the equations in using the template Notice I used m and s , for the letters easier for me Press EXE	And the answers are there for me M = 21.5689- μ S=5.1257- σ	

Binomial Distribution

How do we find the probability when it involves binomial distribution?
Example
Assume that 30% of employed women living in a large city have never being married. If 10 employed women are selected at random, find the probability that at least 7 of them have never been married.

			
Go to statistics Go to calc Then Distribution	This is what you will see once again tap on the bottom and resize	Now select Binomial CD	This is what you will see

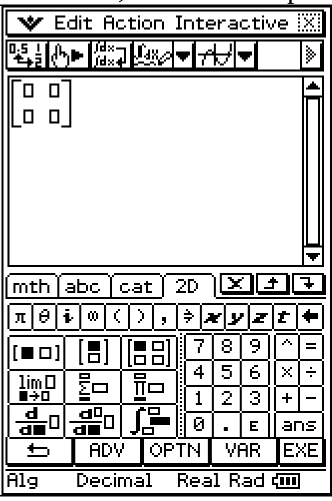
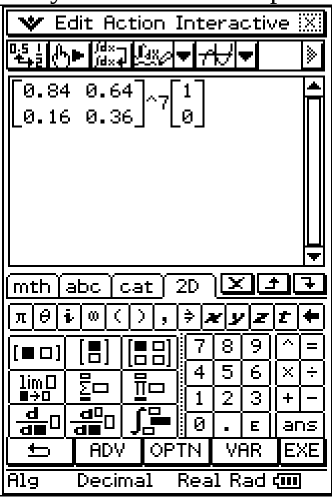
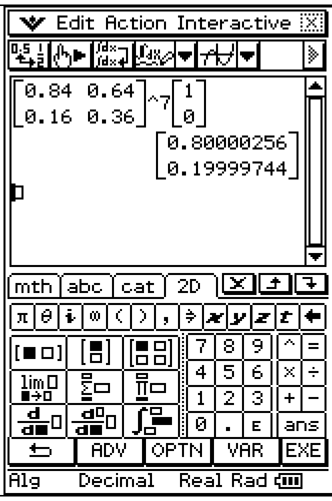
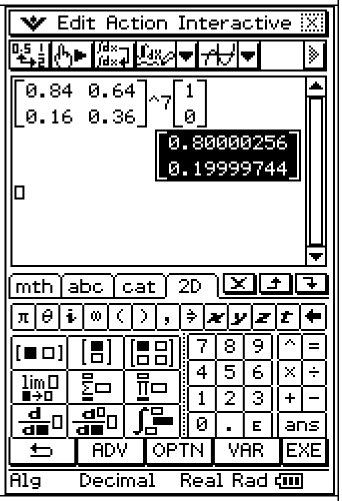
			
Now input the information concerning the question Lower-7 Upper-10 Numtrial-10 Pos-0.3 Now press Next	This is the answer - 0.010592		

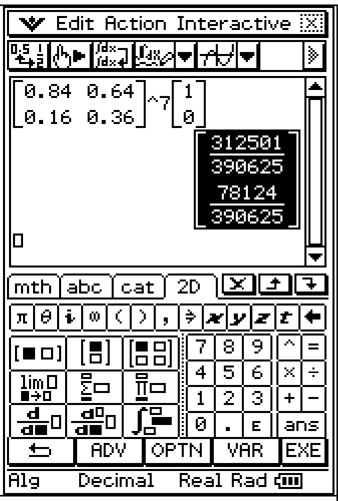
Transition Matrix Probability

How do we calculate non independent probabilities using a transition matrix?

Example

If successful on her first goal attempt, the probability of netballer Lisa scores on her next attempt is 0.84. If she is unsuccessful, then the probability that her next attempt is successful is 0.64. Given that her first attempt was successful, calculate the probability that her 8th attempt is successful?

			
Go to Main application And use the matrix template in 2D Cal	Input the information	And the answer is there You could click on the answer and get the exact if it possible	Highlight the answer and then click on fraction-decimal key

	Successful is 0.8		
The answer is there			

The end of the course, now let’s work on our exam techniques and practice practice practice…….

Quote

“All things good to know are difficult to learn.”- Greek Proverb