Updated: 6 February 2011

Mathematical Methods

Functions and Relations

A quick review of function and relations

V3

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Functions and Relations

Lesson: 1- Interval definitions

Functions

SET OF NUMBERS

A set is a collection of objects. The objects in a set are called 'elements' or members of the set.

$x \in A$ *means* $\rightarrow$ x is a member of the set A
$\rightarrow$ Belongs to A
$\rightarrow$ Is in A

$x \notin A$ $\rightarrow$ x is not an element of set A

If A and B are 2 sets then A $\cap$ B $\rightarrow$ intersection of A and B
$\rightarrow$ Elements common to both the two sets

If A and B have no elements in common then $\rightarrow$ we say that A and B are disjoint or call the set an empty set and use the symbol ϕ

If A U B means union of set A and set B → set of elements are in either set A or set B

A / B = {x: x $\in$ A, x $\notin$ B}

<u>Examples 1</u>

A= {1, 2, 3, 7} and B= {3, 4, 5, 6, 7}

Question 1: Find A $\bigcap$ B

Solution
$\rightarrow$ Find the elements common to both sets A and B

$\rightarrow$3, 7

So the answer is A $\bigcap$ B= {3, 7}

Question 2: Find A $\bigcup$ B

Solution:
Here we want to find elements that are in **A or B**
$\rightarrow$ 1,2,3,4,5,6,7

So the answer is A ⋃ B= {1, 2, 3, 4, 5, 6, 7}

Question 3: A/B

Solution:
A / B = {x: x ∈ A, x ∉ B}
A/B= {1, 2}

Review of set notation

A = {1, 2, 3, 7}

A → Set A

1, 2, 3, 7 → Elements or members of set A

NOTATIONS

Symbols	Meaning
R	Real Numbers: the set of real numbers, namely everything
N	Natural Numbers: 1, 2, 3, 4
Z	Integers: -2,-1, 0, 1, 2, 3, 4, 5….
Q	Rational Numbers of the form $\frac{p}{q}$, where both numbers are integers and q is not equal to 0 Examples: $\frac{1}{2}$, $\frac{-3}{7}$,
Q'	Irrational Numbers, numbers that are not rational: π, $\sqrt{3}$,…..

Intervals Notations

Symbols	Meaning
{ x; 0 < x < 1}	means the set of all real numbers between 0 and 1, not including 0 or 1
{x: x> 0, x rational}	means the set of all positive rational numbers greater than 0
A / B	= {x: x ∈ A, x ∉ B}
B / A	= {x: x ∈ B, x ∉ A}
>	means is greater than
≥	means is greater than or equal to
<	means is lesser than
≤	means is lesser than or equal to
(a, b)	→ {x: a < x < b}

(a, b]	→ {x: a < x ≤ b}
(a, ∞)	→ {x: a < x}
(-∞, b)	→ {x: x < b} → the set of x such that x is less than b
[a, b]	→ {x: a ≤ x ≤ b}
[a, b)	→ {x: a ≤ x < b}
(-∞, b]	→ {x: x ≤ b}

Examples 2

[-2, 3] → {x: -2 ≤x ≤ 3}

(-3, 4] →{x: -3 < x ≤ 4}

(-∞, 4) →{x: x < 3}

(-2, 4) →{x: -2 < x <4}

Probably a good idea sometimes to use a number line to see which numbers are included .

PROBLEM - CHAPTER 1A

In summary remember

(a, b) → (a, b) is called an ordered pair

→ the ***second element***, b

→ the ***first element***, a of the ordered pair

Relation = A set of ordered pairs

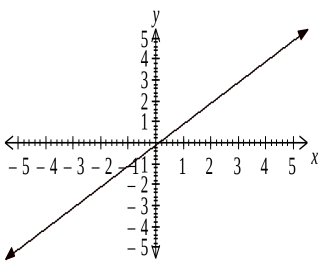
Example 3

S= {(1, 1), (1, 2), (3, 4), (5, 6)}

Domain of S = is the set of all **first elements**, of S, {1, 3, 5}

Range of S = is the set of all **second elements** of S, {1, 2, 4, 6}

Normally we encounter this notation when we are plotting graphs for example notice the following:



This is the graph of y = x

What is its domain?

All real values – R

What is its range?

All real values- R

Now we could of course write down all the values of x that hold true for the above graph, but that would take a long time and is quite burdensome, so we can use another notation, namely

$$\{(x, y): y = x, x \in R\}$$

Now pay particular attention to the meaning of the symbols we have used. Lets us look the notation carefully

{ (x, y) : y = x, x ∈R }

x is element of real numbers

y is given by the rule y=x

such that

the set of ordered pairs (x , y)

This is how it read: the set of all ordered pairs (x , y) such that y is given by the rule y = x, where x is an element of the real numbers.

Quite a mouth full! Anyhow basically it just defines the equation of the graph at tells how what the domain or the range of values that are valid.

Lesson: 2- Functions Beginning

A function is a relation such that 2 ordered pairs do not have the same first element.

So for example if we had the following two ordered pairs: (1, 3) and (1, 4), we can see clearly that this is not a function, since both ordered pairs have the same first element namely 1

How do we work out if a graph is a function?

If a vertical line cuts the graph only once then it is a function!

Just before we precede with some more on this topic just a quick review:

Domain – Set of all possible x values

Codomain- is the set of all possible y values which could be used

Range- is the set of all y values that are obtained

A function, f, can be written in set notation

$$\{(x,y): y = 2x\}$$

or it can be written in another notation called mapping notation

f: X→R where f(x) =

codomain

the rule of the function in terms of x

Domain

Name of the function

Example 1

Write in mapping notation the function written in set notation $\{(x,y): y = 2x\}$

Solution: f: R→R where f(x) =2x

Example 2

If $f(x) = 2x + 1$ find f (3), f (-2), f(x-1)

Solution:

f (3)	= 2(3)+1	f(-2)	= 2(-2)+1	f(x-1)	= 2(x-1)+1
	=6+1		= -4+1		=2x-2+1
	=7		=-3		=2x-1

Example 3

Consider the function f(x) = 2x-4 for all x $\in$R

a) Find the value of f(2), f(-1), f(t)

Solution:

f (2)	= 2(2)-4	f(-1)	= 2(-1)-4	f(t)	= 2(t)-4
	=4-4		= -2-4		=2t-4
	=0		=-6		

b) For what values of t is f (t) =t

f(t)= 2(t)-4= t

→2t-4=t

→t-4=0

→t=4

c) For what values of x is $f(x) \geq x$?

$$2x-4 \geq x$$
$$x-4 \geq 0$$
$$x \geq 4$$

(Remember be very careful when dealing with inequalities since the sign changes if you divide by or multiply by a negative number. If you are unsure use the number line to work out which way the inequality goes)

PROBLEM- CHAPTER 1B

Review

- Relation is any set of ordered pairs (x,y)
- Can be written as a set of points i.e. (1,2), (2,5), (3,7)
- Can be written as an equation or rule i.e. y= $2x^2-4x+1$
- Or it can be shown on a graph

Domain- is the set of first numbers in the ordered pair→ usually x
Range- is the set of second numbers in the ordered pair →usually y

Function

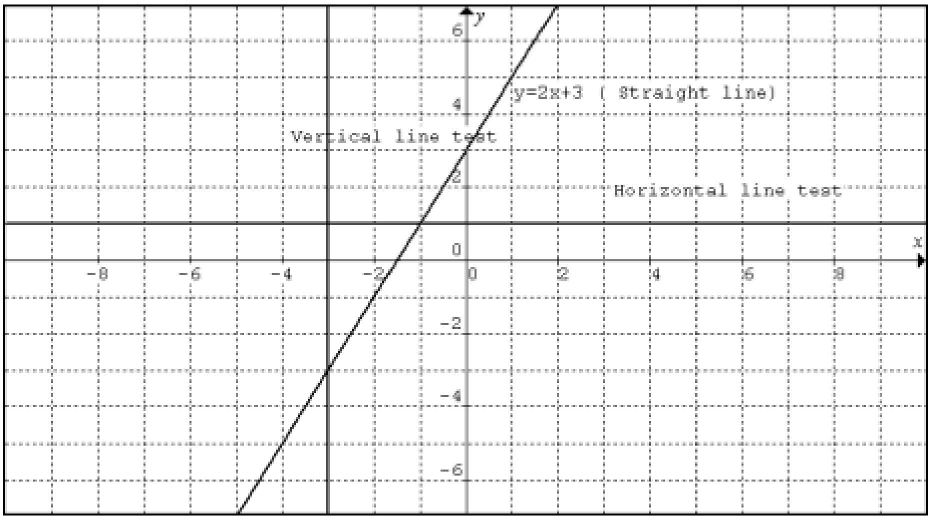
→ it is a relation where no 2 ordered pairs have the same first element

→The graph of a function will satisfy the vertical line test and cut the graph vertically once only

Lesson 3: One to one functions

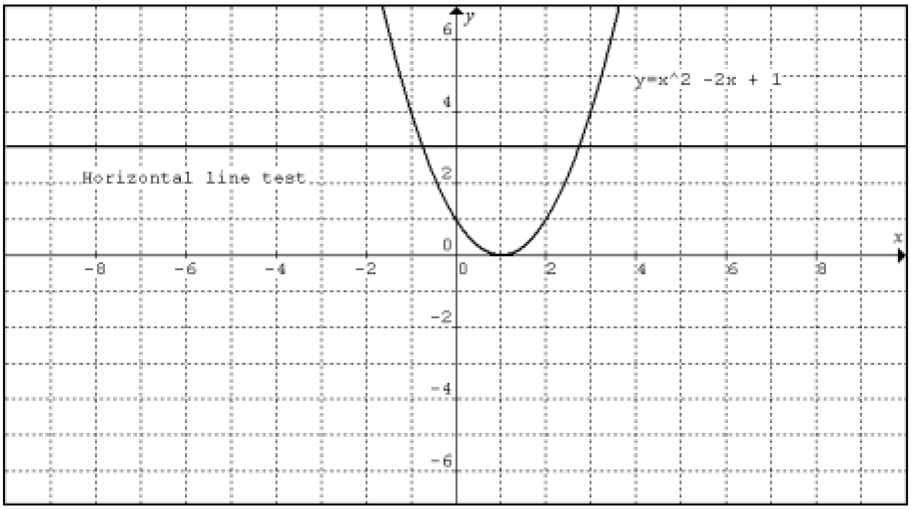
One to one functions: These functions will satisfy both a vertical and horizontal test as no 2 ordered pairs will have the same first or second element.

Only one to one function can have an inverse. A function which is not one to one is many to one.



Notice in the above graph that the vertical line test cuts only once. If you drew a horizontal line it also would only cut the graph once only.

While notice the following graph



The above graph is not a one to one function since it satisfies the vertical line test but fails the horizontal line test. However we could restrict its domain thus making it a one to one function, and that is the reason we spoke about domains and ranges before.

So in the graph above which is $y = x^2 - 2x + 1$, if we restrict the values of x say from $x \geq 1$, then the graph does satisfy both requirements and we can then find a inverse function!

In a few moments we will discuss how to find the inverse but for now we will look at some other properties of functions.

ODD AND EVEN FUNCTIONS

Odd functions have the following property $f(-x) = -f(x)$
Even functions have the following property $f(-x) = f(x)$

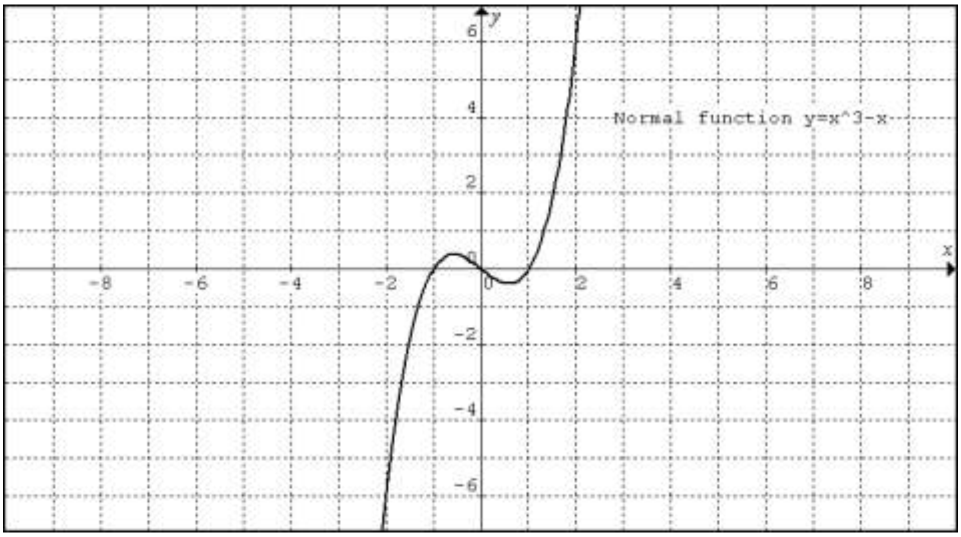
Example 1

$f(x) = x^3 - x$

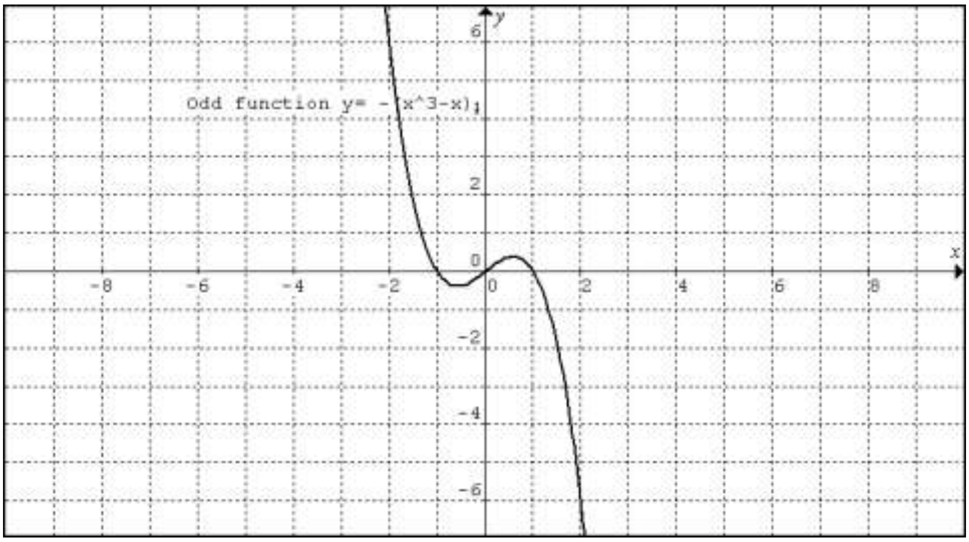
$f(-x) = (-x)^3 - (-x)$
$\rightarrow -x^3 + x$
$\rightarrow -(x^3 - x)$
$= -f(x)$

Thus demonstrating that this function is an odd function.

How do odd functions look when sketched?



Notice now the odd function and see if you can see what has changed?

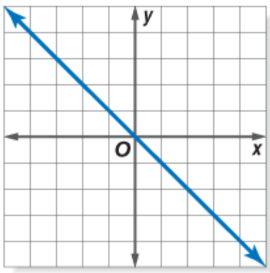


Examples

Example 2

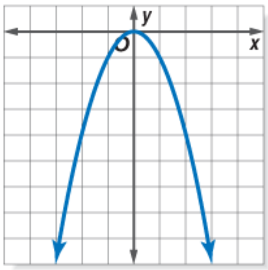
State the domain and range of each relation.

a.



It appears from the graph that all real numbers are included in the domain and the range.

b.



It appears from the graph that all real numbers are included in the Domain. The range includes all negative numbers and zero.

Example 3

State the domain and range of each relation. Then state whether the relation is a function.

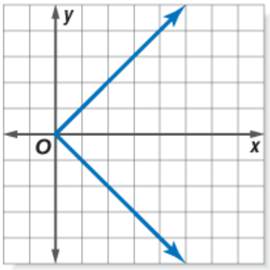
a. {(-1, 2), (0, 4), (1, 3)}
The domain is {-1, 0, 1} and the range is {2, 3, 4}. Each element of the domain is paired with exactly one element of the range, so this relation is a function.

b. {(-3, 1), (-2, 5), (-3, 4), (1, 2)}
The domain is {-3, -2, -1} and the range is {1, 2, 4, 5}. In the domain, -3 is paired with two elements of the range, 1 and 4. Therefore, the relation is not a function.

Example 4

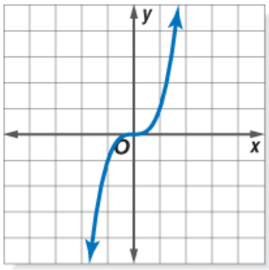
Determine if the graph of each relation represents a function. Explain.

a.



No, the graph does not represent a function. Values of $x > 0$ are paired with two elements of the range. Therefore, it fails the vertical line test.

b.



Every element of the domain is paired with exactly one element of the range. Thus, the graph represents a function.

PROBLEM - CHAPTER 1C

Lesson 4

INVERSE FUNCTIONS

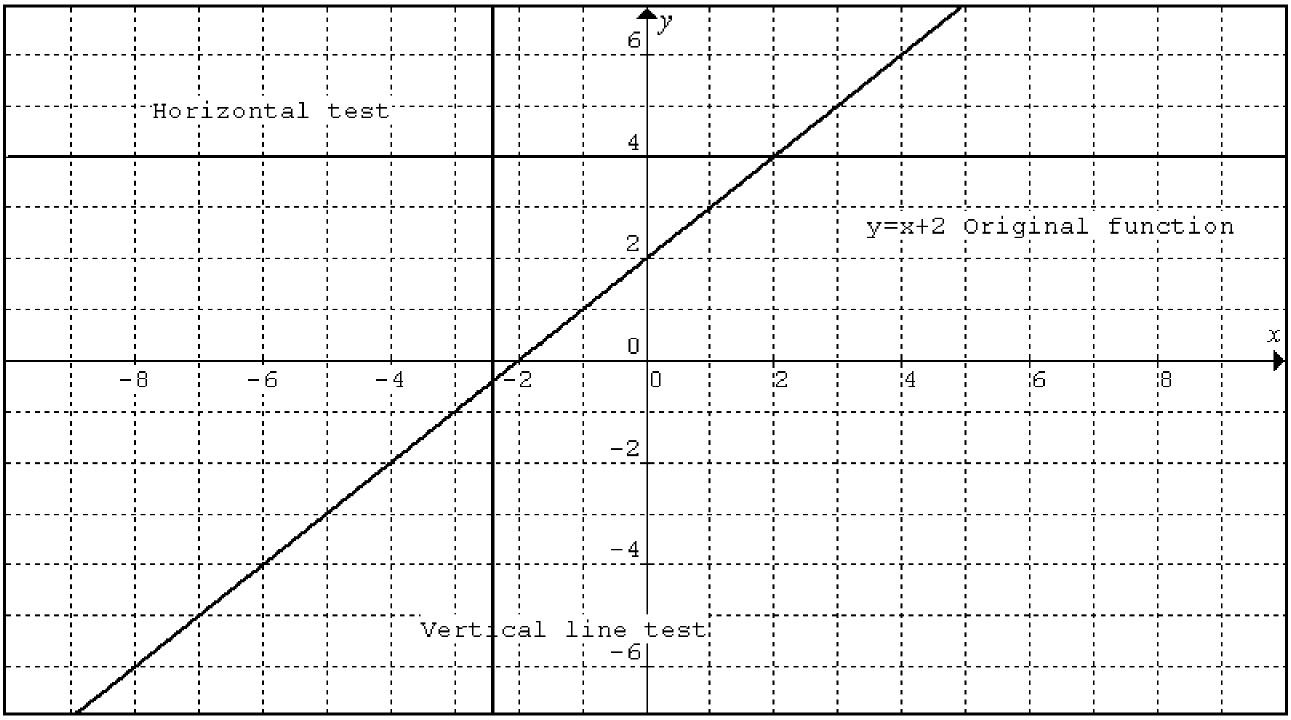
If a function is a one to one function then a new function denoted by the symbol $f^{-1}(x)$, called the inverse function can be found.

All you have to do to find the inverse function is swap x and y values and then make y the subject, and you have the inverse function. (if the function is f(x) just call that y for the sack of simplifying the process!)

Lets us take a simple example to show you how this is done.

Take the function $y = x + 2$

If you sketch this graph you will see immediately that this function satisfies the vertical and horizontal line test thus making it a one to one function and therefore allowing us to find an inverse of the original function.



Now how do we find the inverse since the original function is a one to one function? Simply by swapping the x and y. But in place of the new y we use the following symbol y^{-1} to indicate that we are finding the inverse function of the original y function.

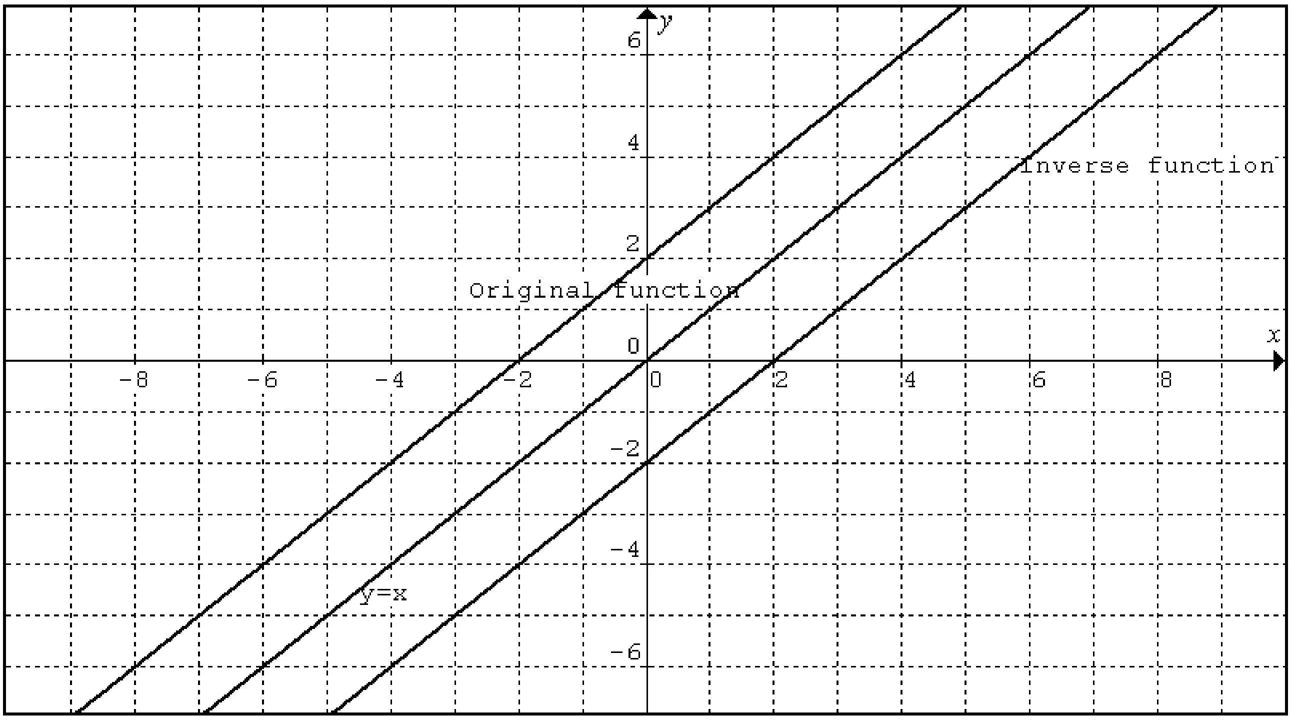
$$y = x + 2$$
$$swap$$
$$x = y^{-1} + 2$$
$$y^{-1} = x - 2$$

The new inverse function is $y^{-1} = x - 2$

Now what is interesting is the following:

The domain of the original function becomes the range of the inverse function.
And the range of the original function becomes the domain of the inverse function.

If we sketch both the original function and the inverse function you will see that they have been reflected along the line y = x (it is as though you placed a mirror along y=x and you got the other function)



So in the language of math's we say the following as occurred

Dom y^{-1} = Ran y

Range y^{-1} = Dom y

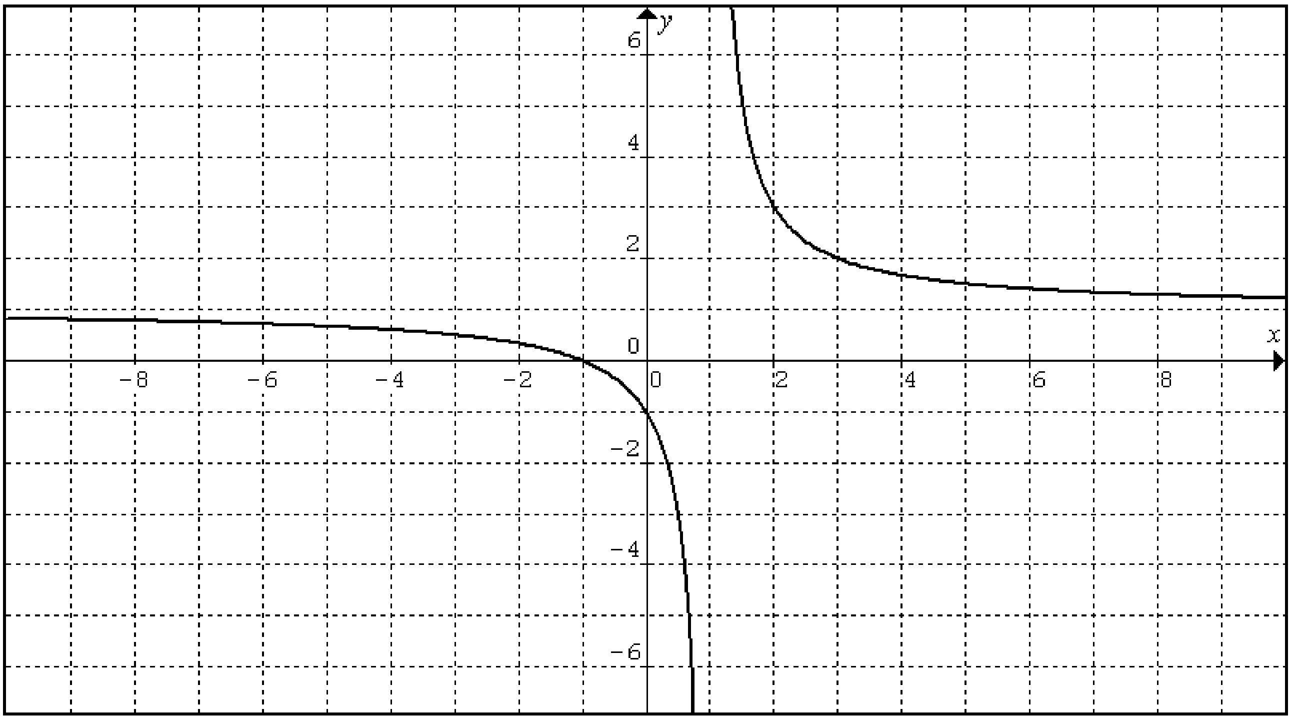
In the above examples the domain and range where the same namely R, but that is not the case normally.

Lets us have a look at a more challenging example which requires a little mathematical manipulation on our part.

Let find the inverse for the function $f(x) = \dfrac{x+1}{x-1}$

For those of us who do not know how this graph looks like lets us use our graphic calculators carefully and see how it looks.

A warning to those who do not use brackets (**USE THEM!!!!**) The graph looks like this



It is obvious that this graph satisfies the vertical line test along with the horizontal line test. This graph as you can see from above contains things called asymptotes (more later).

Now lets us find the equation of the inverse: by swapping x and y values! (For the sake of expedience we shall call f(x) = y) (And you will notice that I use just y instead of y^{-1})

$$
\begin{aligned}
&y = \frac{x+1}{x-1} \\
&\quad swap \\
&x = \frac{y+1}{y-1} \\
&x(y-1) = y+1 \\
&xy - x = y+1 \\
&xy - y = 1+x \\
&y(x-1) = 1+x \\
&y = \frac{1+x}{x-1}
\end{aligned}
$$

Now you would had notice that when I swapped y and x I should had put in the place of y the new function y^{-1} , the reason I did not do this is so not to get confused with a negative power floating around. I prefer to insert it at the end.

What does the inverse look like? The same as before! Strange ah that is why math's is weird and wonderful.

PROBLEM - CHAPTER 1G

Lesson: 5-Modulus functions

Just before we get on with linear lets us discuss the modulus function

The modulus function or otherwise known as the absolute value of a real number *x* is defined by the following

$$x = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

It may also be defined as $|x| = \sqrt{x^2}$

This function $|x|$ has the following properties

$|ab| = |a| \bullet |b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a+b| \leq |a| + |b|$ If a and b are both non negative or both non positive then equality

If $a \geq 0$ then $|x| \leq a$ is equivalent to $-a \leq x \leq a$

If $a \geq 0$ then $|x-k| \leq a$ is equivalent to $k-a \leq x \leq k+a$

Absolute Function

The definition of |x|

The absolute value of *x* is written as |x|. It is defined by the following:

$$|a| = \begin{cases} (a) & \text{if } a \geq 0 \\ -(a) & \text{if } a < 0 \end{cases}$$

|x| can be thought of as the distance that *x* is from zero. For example, the distance that 5 is zero is 5, whereas the distance that -3 is from zero is 3.
So we can then say that |5| = 5 whereas |-3| = 3
Sometimes |x| is referred to as magnitude of *x*, or the modulus of *x*, which can be thought to roughly mean the size of *x*

Graphs of modulus

There are essentially a few ways to sketch modulus functions, namely we can use our graphic calculators (using the graph) or we can go from the definition method.
Lets us examine the definition method
Example 1: Sketch the graph of $y = |x+3|$

Solution: Let us use the definition

Remember the definition $|a| = \begin{cases} (a) & \text{if } a \geq 0 \\ -(a) & \text{if } a < 0 \end{cases}$

In this case we have $a = x+3$
So using the definition we have the following

$$|x+3| = \begin{cases} (x+3) & \text{if } x+3 \geq 0 \\ -(x+3) & \text{if } x+3 < 0 \end{cases}$$

It pays to use brackets often, so in the above case we would have the following:

$$|x+3| = \begin{cases} (x+3) & \text{if } x+3 \geq 0 \\ -(x+3) & \text{if } x+3 < 0 \end{cases}$$

Now we solve each of the above expressions separately.

$$y = x+3 \text{ if } x+3 \geq 0$$

Now to solve this above equation we will have to remember how to deal with inequalities.
Remember the inequality changes if we divide or multiply by a negative number!
So we solve the inequality

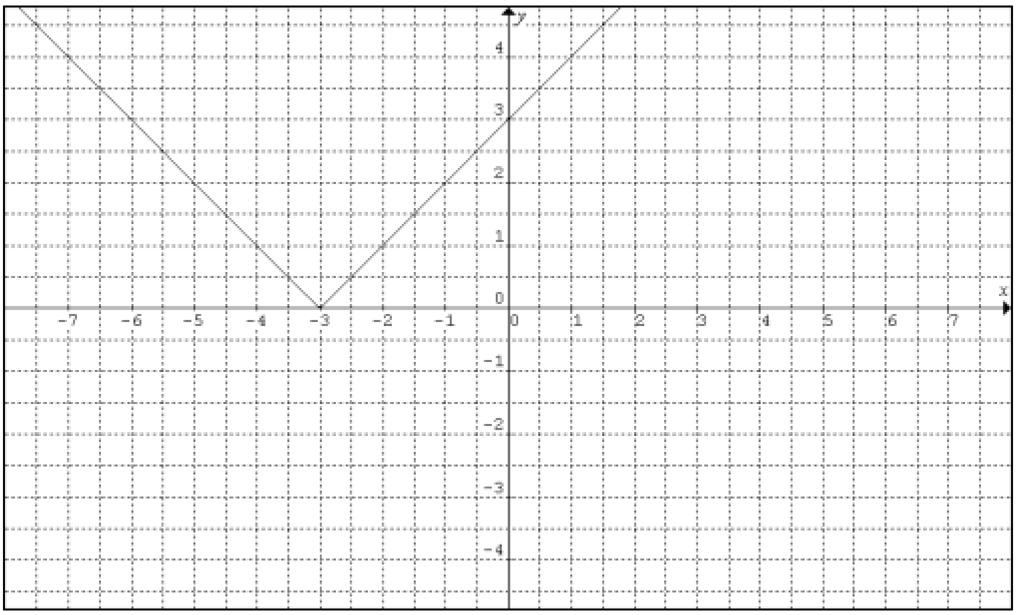
$$x+3 \geq 0$$
$$x \geq -3$$

So this equation looks like this $y = x+3 \quad x \geq -3$

Now we look at the other part of the expression

$$x+3 < 0$$
$$x < -3$$

So this graph would be $y = -x - 3$ for $x < -3$
Now we can sketch this graph



Notice how the graph is positive for all values of x

Now we could have just used our graphics calculators and we would had obtained the above graph quickly, however it is important to be able to do the maths.
Let us look at a few more examples on using modulus functions.
Example 2: $y = |x + 2|$

We use the definition of $|a| = \begin{cases} (a) & if\ a \geq 0 \\ -(a) & if\ a < 0 \end{cases}$ to see how to sketch this modulus function

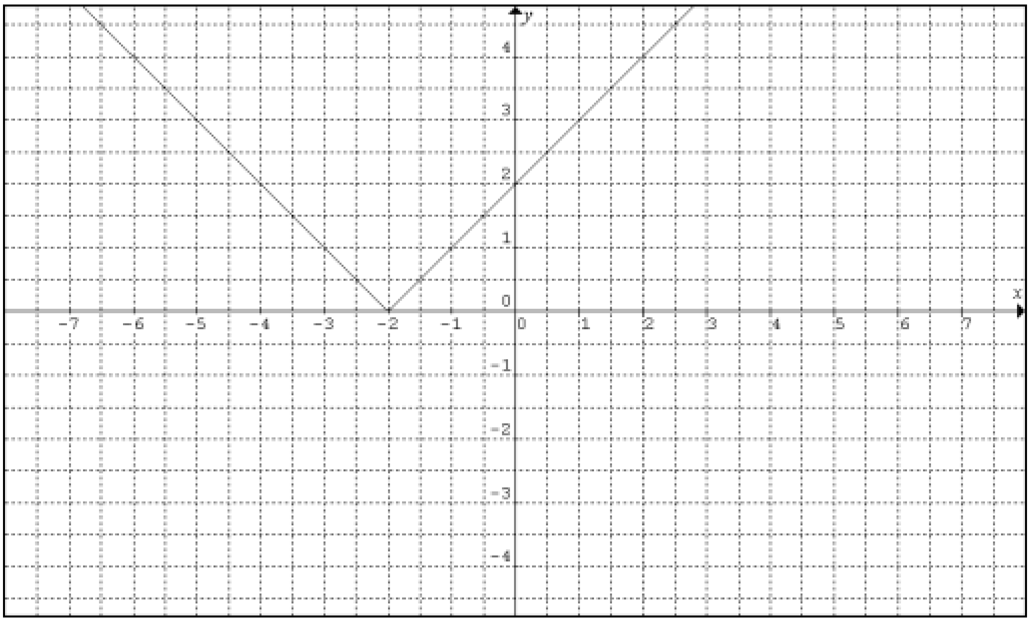
Here in the place of $a = x + 2$

So we have the following $|x+2| = \begin{cases} (x+2) & if\ (x+2) \geq 0 \\ -(x+2) & if\ (x+2) < 0 \end{cases}$

Let us break it down into two expressions

$y = x + 2$ if $(x+2) \geq 0$, which basically means $x \geq -2$

$y = -(x+2) \rightarrow y = -x - 2$ Which applies for the following $(x+2) < 0 \rightarrow x < -2$



Notice how the next graph looks a little different; in the sense the modulus signs only cover the x values

Example 3: Sketch $y = |x| - 4$

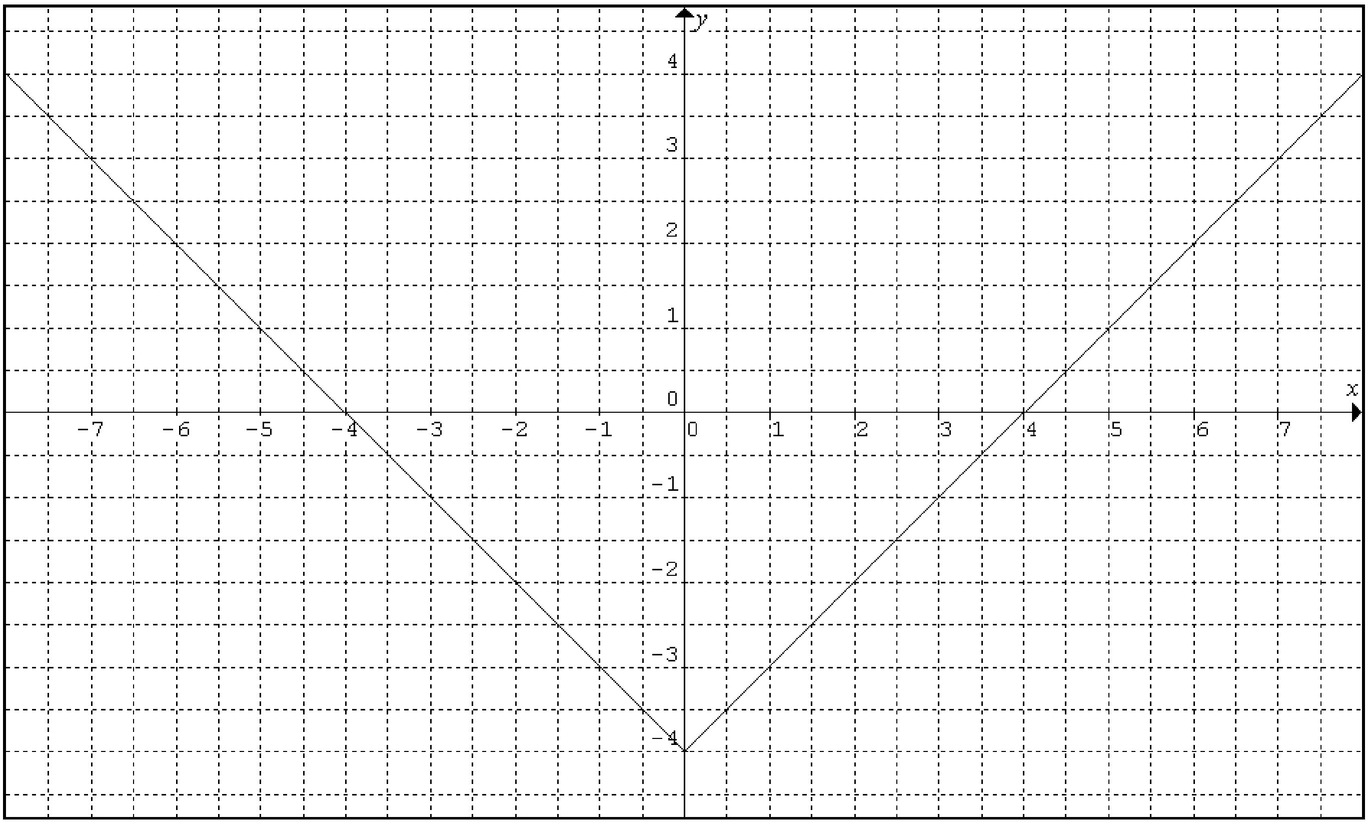
Let us use basic principles

Remember the definition $|a| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$

$$|x| - 4 = \begin{cases} x - 4 & if\ x \geq 0 \\ -(x) - 4) & if\ -x < 0 \end{cases}$$

Now the graph of $y = x - 4$ is for $x \geq 0$

While the graph of $y = -x - 4$ if for $-x < 0$



Try the following questions to hone your skills
Sketch the graph of each of the following modulus function. Make sure you include the domain and range of the function,

a) Sketch the graph of $y = |x + 5|$
b) Sketch the graph of $y = |x| + 2$
c) Sketch the graph of $y = |x - 1|$
d) Sketch the graph of $y = |x - 1| + 5$
e) Sketch the graph of $y = |x + 3| - 5$
f) Sketch the graph of $y = |2x - 1| + 5$
g) Sketch the graph of $y = 4 - |x|$
h) Sketch the graph of $y = -|x - 2| + 1$

More difficult questions regarding modulus functions

How do we sketch the following graph? $y = |x^2 + x - 2|$
Let us use the definition of modulus

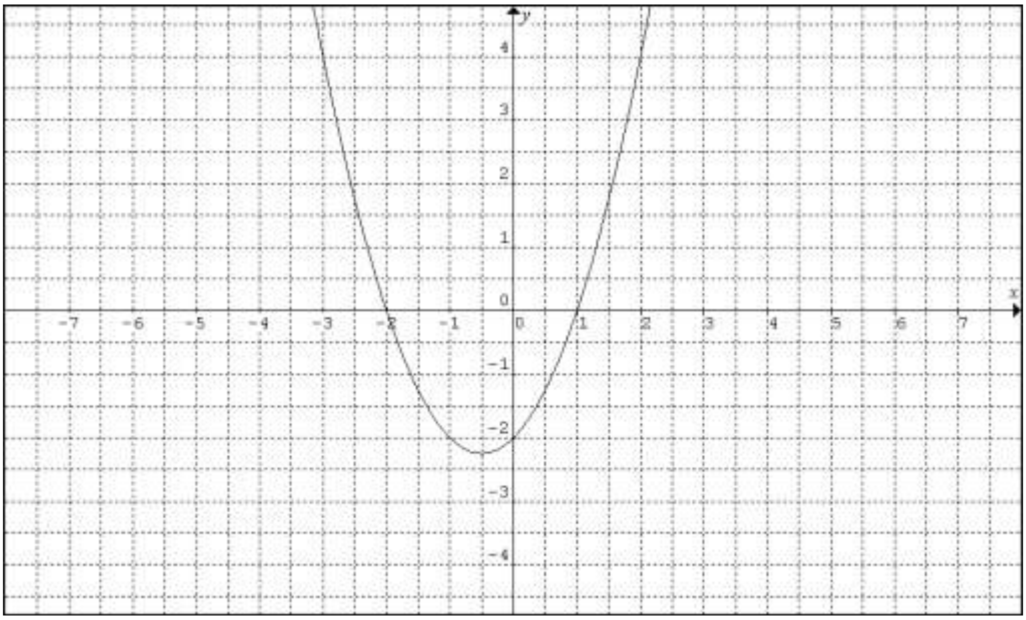
$$|a| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$$

$$|x^2 + x - 2| = \begin{cases} (x^2 + x - 2) & if\ x^2 + x - 2 \geq 0 \\ -(x^2 + x - 2\ ) & if\ x^2 + x - 2 < 0 \end{cases}$$

Now this is where it gets difficult

$$x^2 + x - 2 \geq 0$$

We can factorise the above quadratic equation, $x^2 + x - 2 \rightarrow (x + 2)(x - 1)$
Now lets us sketch the normal graph of y= $x^2 + x - 2$



From the above graph we can see that the x-intercepts are x= -2 and x=1
Now let us consider the two expressions to see when they are true

$$x^2 + x - 2 \geq 0$$
$$(x + 2)(x - 1) \geq 0$$

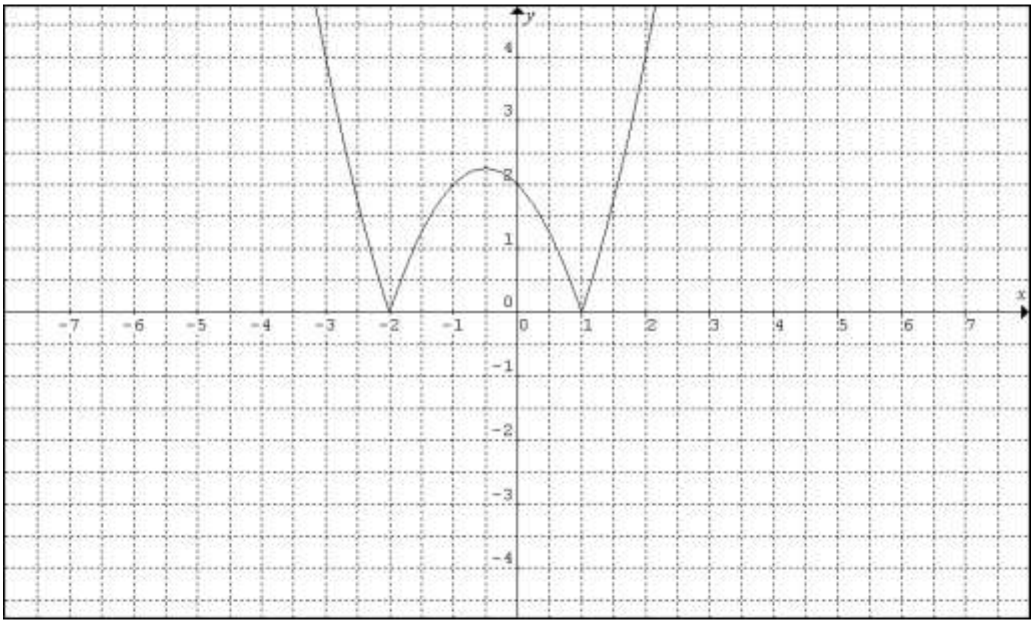
To get the above expression to be positive both brackets must be positive or both brackets must be negative, therefore $x \leq -2 \; or \; x \geq \; 1$

The second expression $-(x^2 + x - 2\,) \;\; if \; x^2 + x - 2 < 0$

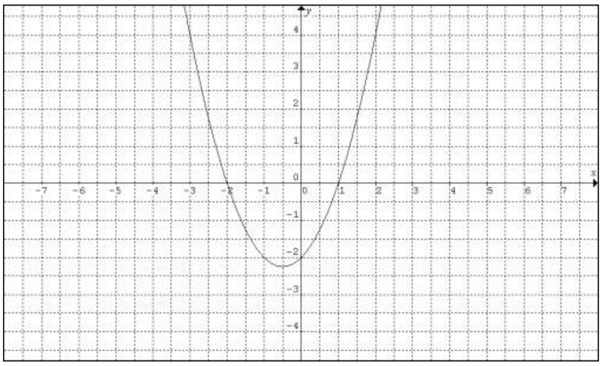
$$(x + 2)(x - 1) < 0$$

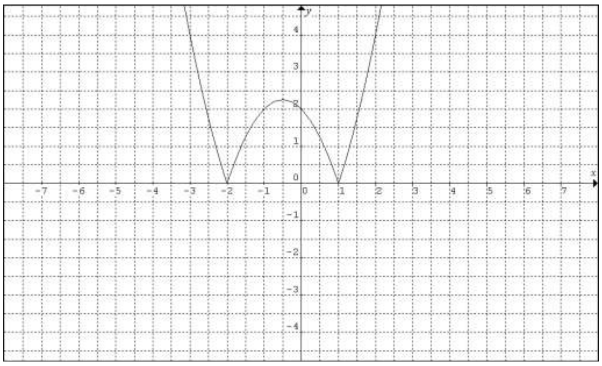
To get the above expression < 0 then x must be between -2 and 1, we write this like (-2, 1)

Then we can sketch the above graph taking into account the domain of the two expressions



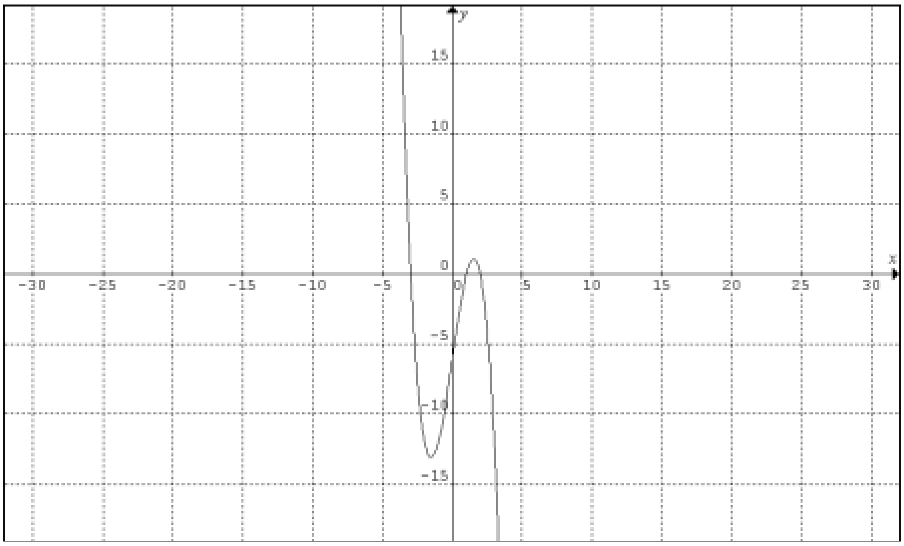
If you like at the two graph side by side notice what has actually happen.

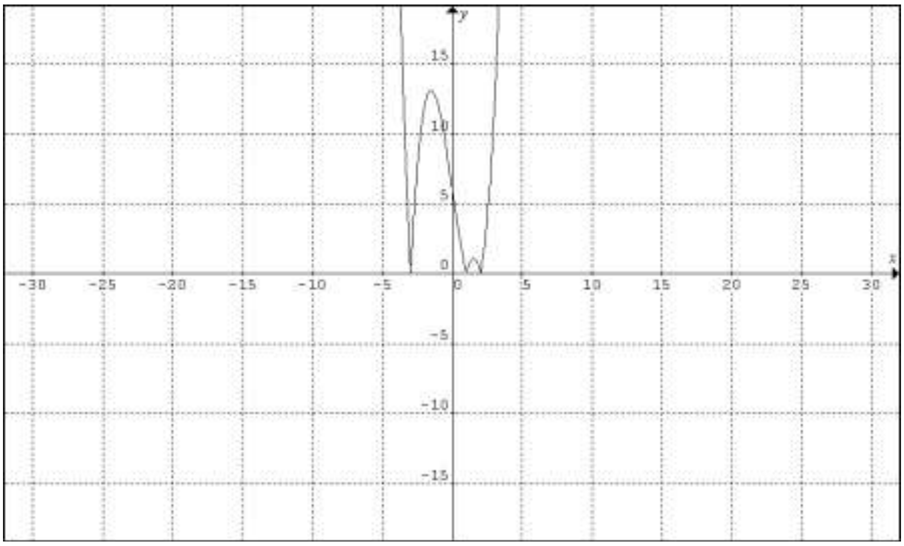




Method 2: Simply sketch the graph y = $x^2 + x - 2$. The absolute value of a positive number is equal to that number, and the absolute value of a negative number is equal to the negative number and is therefore positive. So we simply sketch the normal graph and reflect in the x-axis the part of the graph that has a negative y value.

Example: Sketch the graph of $y = |(2 - x)(x - 1)(x + 3)|$
Let's have done it the fast way





Skill builder

a) Sketch the graph of $y = |x^2 + 3x - 10|$
b) Sketch the graph of $y = |x^2 - 2 - 8|$
c) Sketch the graph of $y = |(x-1)(x+1)(x+3)|$
d) Sketch the graph of $y = |(x+1)(x+3)(x+4)|$
e) Sketch the graph of $y = |(x-1)^2(x-3)|$
f) Sketch the graph of $y = |(x+2)^2(x-2)|$

PROBLEM - CHAPTER 1D

Lesson 6-Composite functions

Composite Functions

What are they?

In the real world, it is not uncommon for the output of one thing to depend on the input of another function. For example the amount of tax we would pay depends on the gross salary the person makes. Such functions are called composite functions.

So a function is performed first and then a second function is performed on the result of the first function, that is what is actually taking place when we composition.

Special terminology

The composite function f ° g, the composition of f and g is defined as follows

$$(f \circ g)(x) = f(g(x))$$

For the above function to be defined or to exist then a certain condition must be met namely

$$Range\ g(x) \subseteq \text{domain } f(x)$$

Example

Let us consider an example and we will see how this works in practice

Consider the following two functions

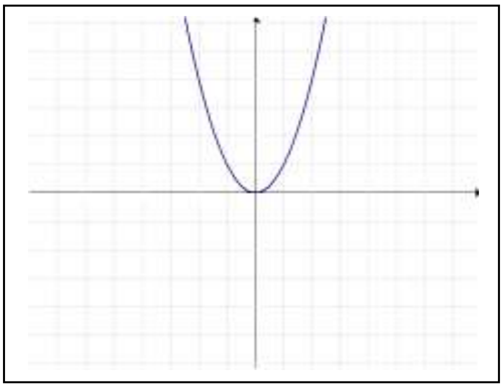
$$f(x) = x^2 \quad \text{and} \quad g(x) = x + 4$$

Say we will like to find the following composite function $f \circ g$ and whether this function exists

Step 1

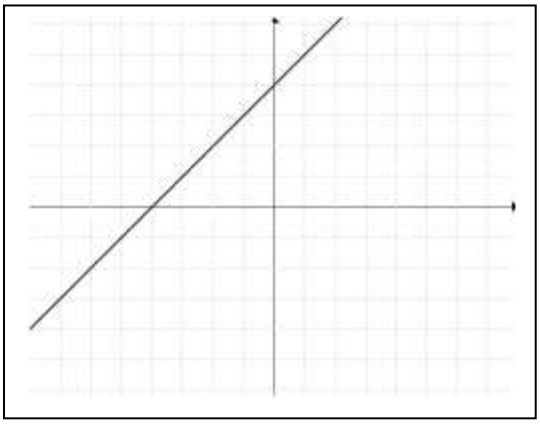
Sketch the two functions and find their respective domains and ranges

$f(x) = x^2$



$f(x) = x^2$
Domain: R - (-∞, ∞)
Range: R- (0, ∞)

$g(x) = x + 4$



Now the graph of
$g(x) = x + 4$
it domain is as follows
Domain : R - (-∞, ∞)
Range: R - (-∞, ∞)

Now the reason we plotted these two graphs is to help us understand the restrictions that must be placed on the domain for the various composite functions to be defined

So let's tackle the first question

Find $f \circ g$

Answer

$(f \circ g)(x) = f(g(x))$

$f \circ g(x) = f(g(x))$

$\rightarrow f(x+4)$

$\rightarrow (x+4)^2$

Now for this function to be defined the condition *Range* $g(x) \subseteq$ domain $f(x)$

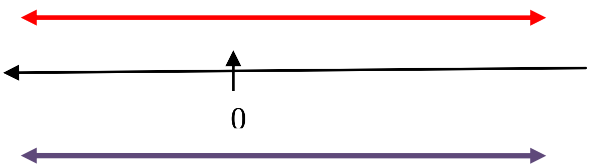
Let's see how this looks

We have the following operation $x \rightarrow g(x) \rightarrow f(x)$

Range g(x): (-∞, ∞)

Domain *g(x)*
Range *g(x)*
Domain : R - (-∞, ∞)
Range: R - (-∞, ∞)

Domain *f(x)*
Range *f(x)*
Domain: R - (-∞, ∞)
Range: R- (0, ∞)



Domain f(x) :(-∞, ∞)

So since range of g(x) is a subset of the domain of f(x) then this composite function exists.

Now let's see if we can find the composite function $g \circ f$

Let's follow the previous steps

$g \circ f(x) = g(f(x))$

$\rightarrow g(x^2)$

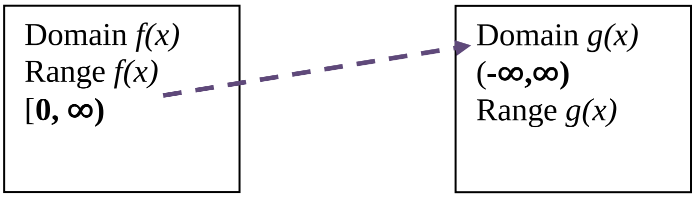
$\rightarrow x^2 + 4$

For this to be defined then *Range* $f(x) \subseteq$ domain $g(x)$

Now this is the most important step as it shows the actual process that is taking place here

$$x \to f(x) \to g(x)$$

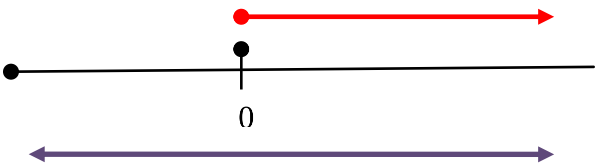
So that gives us the following



Now remember that

$Range\ f(x) \subseteq$ domain $g(x)$

Range f(x): [0, ∞)



Domain g(x) :(-∞, ∞)

The range of f(x) is a subset of the domain of g(x), so this composite function is defined.

Important Points to Remember

For $f \circ g$

$domain\ f \circ g = domain\ f$

$range\ f \circ g = range\ g$

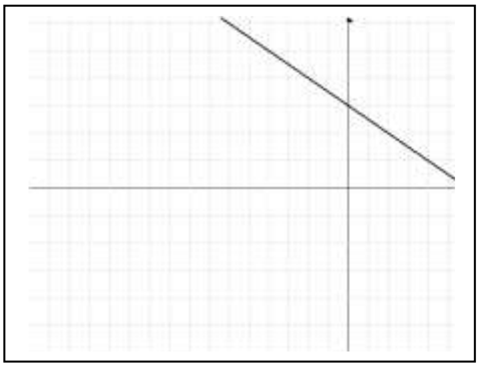
Difficult Example

Consider the following two functions $f:\{x:x\le 3\}\to R,\ f(x)=3-x$ and $g:R\to R, g(x)=x^2-1$

a) Show that $f\circ g$ is not defined

Let us sketch both graphs and work out their domains and ranges before we answer the question

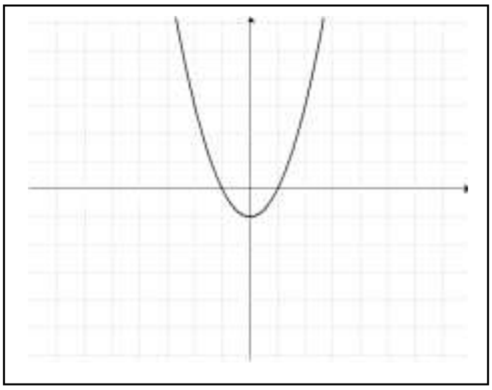
$f:\{x:x\le 3\}\to R,\ f(x)=3-x$



$f:\{x:x\le 3\}\to R,\ f(x)=3-x$

it domain is as follows
Domain: (-∞, 3]
Range: [0, ∞)

$g:R\to R, g(x)=x^2-1$



$g:R\to R, g(x)=x^2-1$

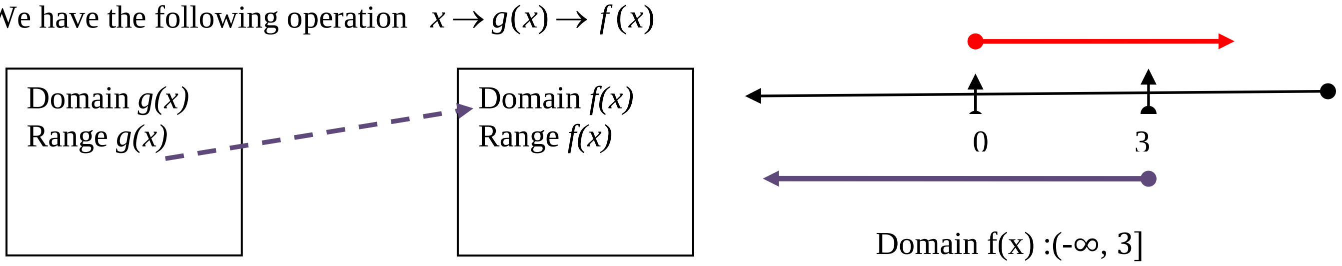
Domain: (-∞, ∞)
Range: [-1, ∞)

Now for $f\circ g$ to be defined the condition $Range\ g(x)\subseteq$ domain $f(x)$

Range g(x): [0, ∞)

Let's see how this looks

We have the following operation $x\to g(x)\to f(x)$



So it is clear that range g(x) is not a subset of the domain f(x). So the composite function $f\circ g$ is not defined. Of course we could define it if we restrict the domain of f to [-1, 3]. Remember $domain\ f\circ g = domain\ f$

Lesson: 7- Linear functions

LINEAR FUNCTIONS

There is no need to spend too much time on these functions as you have seen them from previous years but lets us just review some of the basic concepts that you should have in mind.

General equation of a straight line $y = mx + c$

m stands for the gradient or another word for gradient is slope= rise/run

c stands for the Y intercept

Sometimes we also write it in another form: ax + by = c (X intercept = c/a) (Y intercept = c/b)

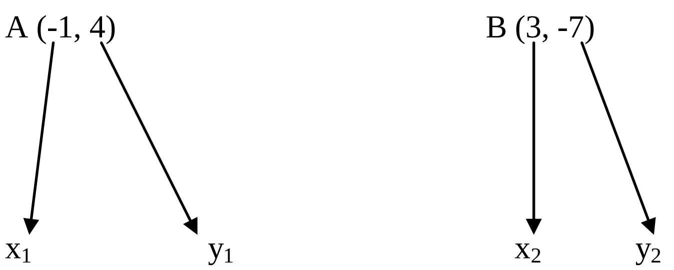
To find the gradient of a straight line joining 2 points we use the following formulae: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Now the good news is that it does not matter which of the 2 points we call (x_1, y_1) or (x_2, y_2)

WARNING: ONCE AGAIN BE CAREFUL OF NEGATIVES SIGNS AND USE BRACKETS!

Example 1

Find the gradient of a straight line joining the following two points: A (-1, 4) and B (3, -7)



$$m = \frac{-7-4}{3-(-1)}$$
$$= \frac{-11}{4}$$

The equation of a straight line passing through point (x_1, y_1) and having a gradient of m is given by the following equation $y - y_1 = m(x - x_1)$

If 2 straight lines are perpendicular to each other then the product of their gradients is -1

$$m_P m_\perp = -1$$

m_p means Gradient of the parallel line while $m_\perp$ means the gradient of the perpendicular line

Length of the line is given by the following formula: AB $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Where A(x_1, y_1) and B (x_2, y_2)

The midpoint of the line joining points A(x_1, y_1) and B (x_2, y_2) is given by the formula:

Mid-point $=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Example 2

Find the midpoint of PB where P (1, 3) and B (10, 17)

Solution:

$$=\left(\frac{1+10}{2},\frac{3+17}{2}\right)$$

Mid-point $=(11/2,20/2)$

$$=(5.5,10)$$

Example 3

Find the equation of the straight line which passes through (1, 6) and is parallel to the line with equation y= 2x+3

Solution:

If this straight line is parallel to the line with equation y=2x+3 that means both these lines have the same gradient and it's so happens that you can see that the line y=2x+3 has m= 2.

So now we have the gradient of the line m=2

Now we use the general equation from above namely:

"The equation of a straight line passing through point (x_1, y_1) and having a gradient of m is given by the following equation $y-y_1=m(x-x_1)$"

$$y-y_1=m(x-x_1)$$
$$y-6=2(x-1)$$
$$y-6=2x-2$$
$$y=2x-2+6$$
$$y=2x+4$$

Example 4

Find the equation of the straight line which passes through (1, 6) and is perpendicular to the line with equation $y=2x+3$

Solution
Everything the same except this time the line is perpendicular to the line with equation $y = 2x + 3$

Here we have to find the gradient using the formula: $m_P m_\perp = -1$

$$m_P m_\perp = -1$$
$$2m_\perp = -1$$
$$m_\perp = \frac{-1}{2}$$
$$m_\perp = -.5$$

Now that we have the gradient we use the $y - y_1 = m(x - x_1)$

$$y - y_1 = m(x - x_1)$$
$$y - 6 = -.5(x - 1)$$
$$y - 6 = -.5x + .5$$
$$y = -.5x + .5 + 6$$
$$y = -.5x + 6.5$$

<u>Example 5</u>

Find the equation of the straight line with m=3 passing through (1, 6)

Solution

$$y - y_1 = m(x - x_1)$$
$$y - 6 = 3(x - 1)$$
$$y - 6 = 3x - 3$$
$$y = 3x - 3 + 6$$
$$y = 3x + 3$$

<u>Example 6</u>

Find the equation of the line that passes through (1,2) and is perpendicular to the line with equation 4x+2y= 10

Solution:

We need to find the value for the line 4x + 2y=10, however you notice it is not in a convenient form so that we could just simply read the value of m!

We must rewrite it into the simple form

$$4x + 2y = 10$$
$$2y = 10 - 4x$$
$$\frac{2y}{2} = \frac{10}{2} - \frac{4x}{2}$$
$$y = 5 - 2x$$
$$y = -2x + 5$$
$$y = mx + c$$
So m= -2

Now we will find the gradient of the perpendicular line using the formula:

$$m_P m_\perp = -1$$
$$-2m_\perp = -1$$
$$m_\perp = \frac{-1}{-2}$$
$$m_\perp = .5$$

Now we use the equation $y - y_1 = m(x - x_1)$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = .5(x - 1)$$
$$y - 2 = .5x - .5$$
$$y = .5x - .5 + 2$$
$$y = .5x + 1.5$$

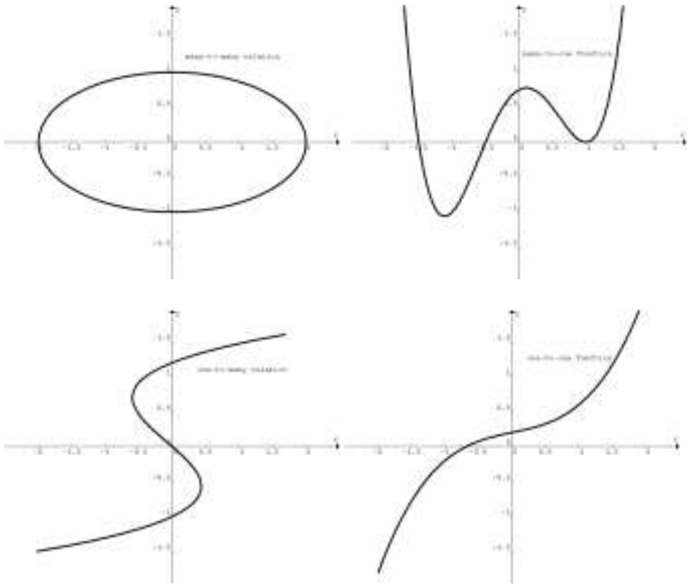
All that work, and unfortunately there are no sort cuts at this moment.

PROBLEM - CHAPTER 2A,2B,2C

Extra comments

Relations and functions

A relation is a set of ordered pairs (points). A function is a relation such that no two points have the same x-coordinate. Use the **vertical line test** to determine whether a relation is a function (cuts through only one point) or not (cuts through more than one point). If it is a function, then it is either a **many-to-one function** or a **one-to-one function**. If it is not a function, then it is either a **many-to-many relation** or a **one-to-many relation**.

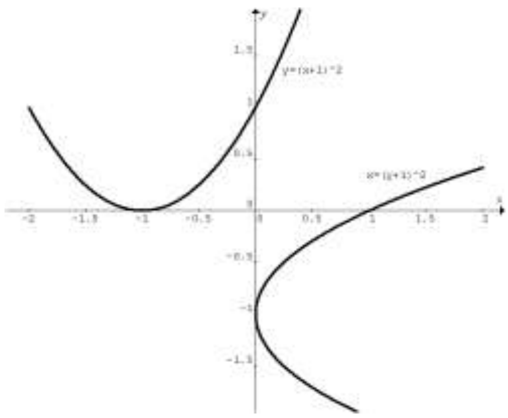


Inverse functions

Every relation has an inverse that may or may not be a function. If a relation is a one-to-many relation, then its inverse is a many-to-one function. If a relation is a one-to-one function, then its inverse is also a function (a one-to-one function). If a relation is a many-to-many relation or many-to-one function, then its inverse is not a function.

Use the **horizontal line test** to determine whether the inverse is a function (cuts through only one point) or not (cuts through more than one point).

Example 1: The following two graphs show the original relation $y=(x+1)^2$ that is a many-to-one function, and its inverse $x=(y+1)^2$ that is not a function.



Example 2: The relations $y=Sin(x)$ for $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, $y=x^2$ for $x\geq 0$ and $y=x^2$ for $x<0$ are one-to-one functions, $\therefore$ their inverses are also one-to-one functions.

If a relation is a function, **function notations** can be used to represent it, e.g. $y=x^2$ for $x<0$, $f:R^-\to R, f(x)=x^2$. Since its inverse $y=-\sqrt{x}$ is also a function, use f^{-1} to denote inverse function, $f^{-1}:R^+\to R, f^{-1}(x)=-\sqrt{x}$.

Inverse functions undo each other

When a one-to-one function *f* and its inverse function f^{-1} are used to form composite function $f^{-1}\circ f$ or $f\circ f^{-1}$, then

$f^{-1} \circ f(x) = x$, $f \circ f^{-1}(x) = x$, i.e. they undo each other.

Example 9: Given $f(x) = (x-2)^3$, find $f^{-1}(x)$. Show that they undo each other.

Equation of *f* is $y = (x-2)^3$, $\therefore$ equation of f^{-1} is $x = (y-2)^3$, i.e. $y = \sqrt[3]{x} + 2$, $\therefore f^{-1}(x) = \sqrt[3]{x} + 2$.

$$f^{-1} \circ f(x) = f^{-1}(f(x)) = \sqrt[3]{f(x)} + 2 = \sqrt[3]{(x-2)^3} + 2 = x$$
$$f \circ f^{-1}(x) = f\left(f^{-1}(x)\right) = \left(f^{-1}(x) - 2\right)^3 = \left(\sqrt[3]{x}\right)^3 = x.$$